EINDHOVEN UNIVERSITY OF TECHNOLOGY **DEPARTMENT OF THE BUILT ENVIRONMENT**

Intermediate test: Design of Structures (7P8X0)

Date: 16/08/2017 NAME (in capital letters):

Time: 09.00-12.00 ID.NR.:

Exercises and response-sheets

Please read the following instructions before writing and submitting the answers to the test.

- Put the answers only in the dedicated boxes and/or in the tables and/or in the figures. The answers must be clearly legible. Other comments are not corrected.
- Draw charts and sketches in the pre-printed figures on scale.
- Only one set of response-sheets is given per student. So think carefully before filling in something.

Do not remove staples!
The last page (coefficient tables) may be removed from the other pages.

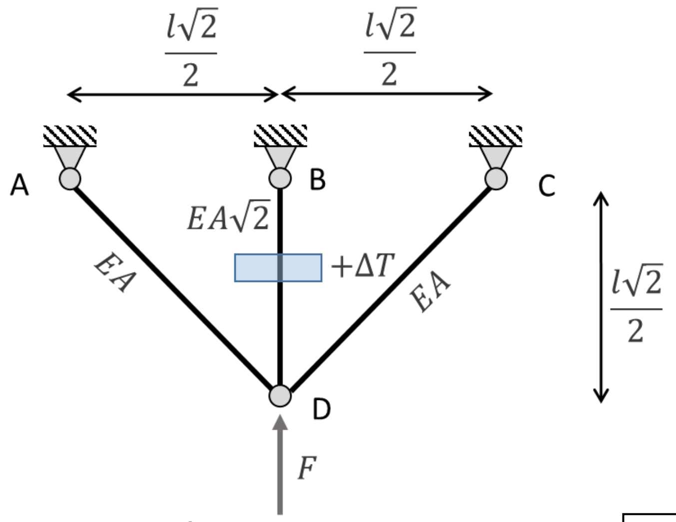
Put the name and ID number of university card at the top of the front page.

Scores for the exercises:
Exercise 1: 15 points
Exercise 2: 20 points
Exercise 3: 15 points

Evaluation of the test: total number of points scored divided by five. The final test counts for 50% of the final grade.

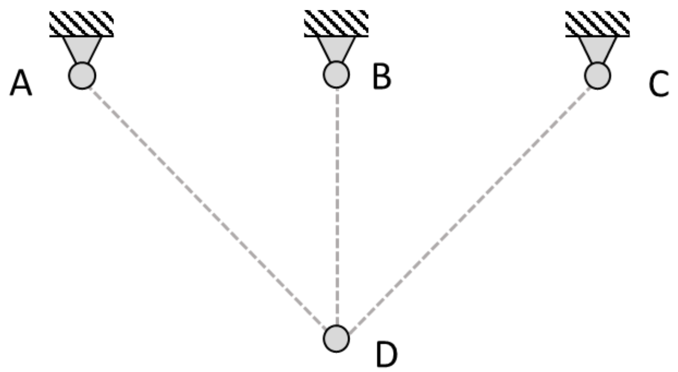
Do not use notebooks, laptop, cell phone etc. A small non-graphic calculator is allowed.

1. Consider the truss structure shown in the Figure below. The supports A, B, C are hinges. Rods AD and CD have axial rigidity EA, rod BD has axial rigidity EA$\sqrt{2}$. The beam is subjected to a **load F**, applied in point D. Moreover, rod BD is subjected to a thermal variation $+\Delta T$. Its thermal expansion coefficient is α. Determine the reaction forces in points A, B and C. Solve the exercise by following the guidelines below.

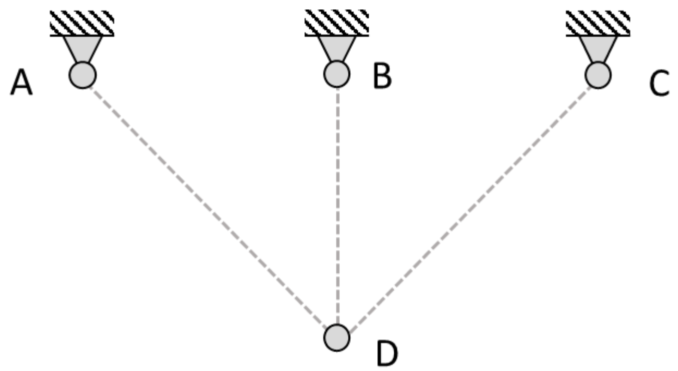


1a. Degree of static indeterminacy of the structure:

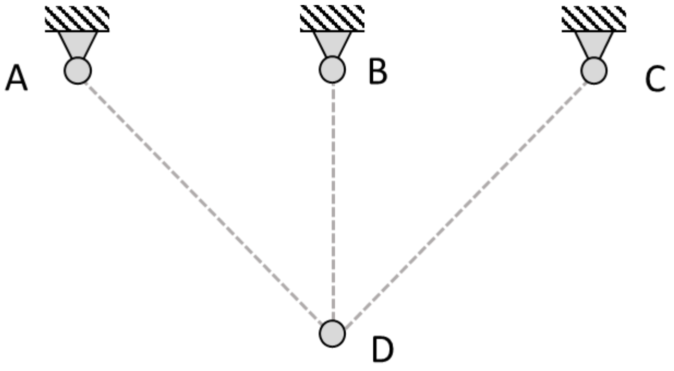
1b. Draw in the Figure below the selected statically determinate equivalent system.



1c. Calculate and write on the Figure the **reaction forces** in case <u>only the external load</u> is acting on the equivalent statically determinate structure. **Draw the corresponding deformation**.



1d. Calculate and write on the Figure the **reaction forces values** in case only the unknown redundant force is acting on the equivalent statically determinate structure. **Draw the corresponding deformation.**



1e. Assuming first that $\Delta T = 0$, i) write the **deformation condition** and ii) the obtained **value of the unknown redundant force**.

i) Deformation condition:

ii) Redundant force:

1f. Write the values of the reaction forces in the structure. Indicate also the direction.

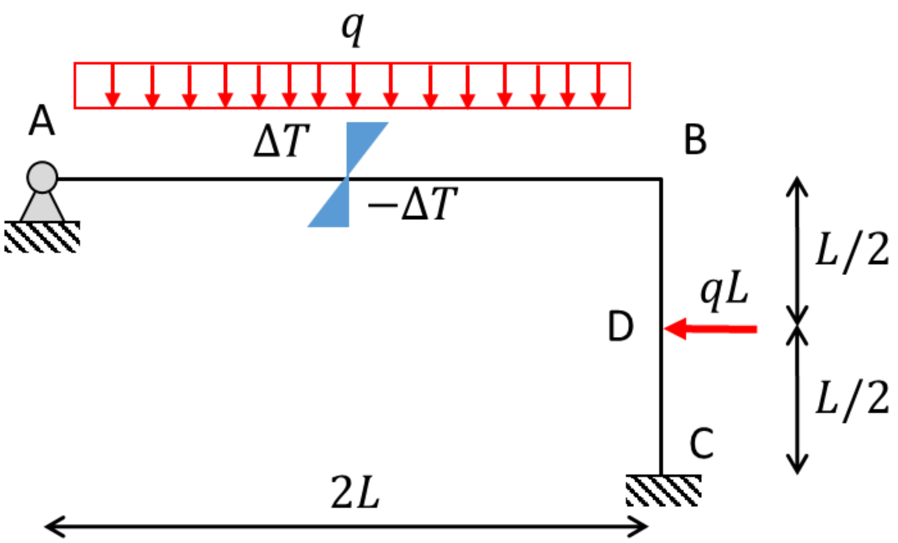
Reaction force in point A		*Direction*
Reaction force in point B		*Direction*
Reaction force in point C		*Direction*

1g. Consider finally also the contribution of the thermal variation ΔT. i) Write the new **deformation condition**; ii) Calculate the **redundant force**.

i) <u>Deformation condition:</u>

ii) <u>Redundant force:</u>

2. Consider frame A, B, C. Part AB is loaded by a **distributed load q**. Moreover, it is subjected to a **temperature variation** of $+\Delta T$ on the top side, and of $-\Delta T$ on the bottom side. In point D, a **concentrated load qL** is applied. The stiffness EI is constant for all the frame. The cross section of the frame is characterized by a height $h = L/10$. The thermal expansion coefficient is α. In the solution of the exercise, assume $\alpha\Delta T = \frac{ql^3}{80EI}$. The change in length of the beams due to normal forces is neglected. **<u>Use the FORCE METHOD</u> to determine the external forces, to draw the internal forces diagrams and to sketch the deformation of the structure.** The coefficient tables are attached at the end of the sheets. Solve the exercise by following the guidelines below.



2a. Degree of static indeterminacy of the structure:

2b. Draw in the Figure below the selected statically determinate equivalent system.

2c. Write the i) **deformation condition(s)** and ii) the obtained **value of the unknown redundant forces/moments**.

i) Deformation condition:

ii) Redundant forces/moments:

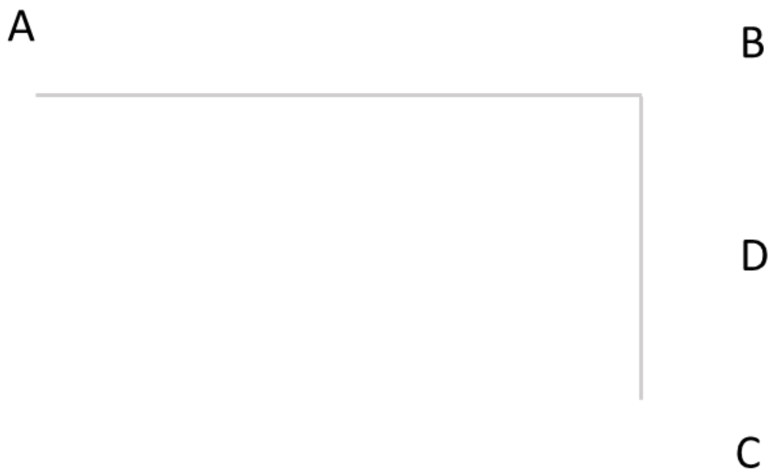
2d. Write the values of the reaction forces in the structure. Indicate also the direction.

Vertical reaction force in point A		*Direction*

Horizontal reaction force in point A		*Direction*
Reaction moment in point C		*Direction*
Vertical reaction force in point C		*Direction*
Horizontal reaction force in point C		*Direction*

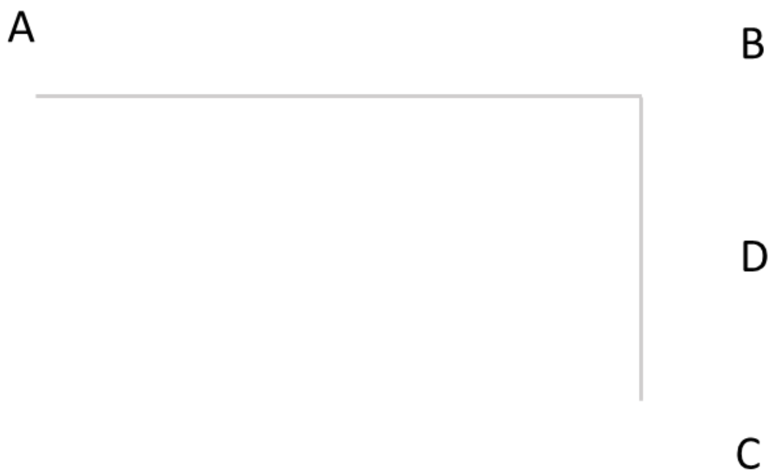
2e. Draw the **internal forces diagrams**.

i) SHEAR



Coordinates of the point along AB in which the shear is equal to zero:

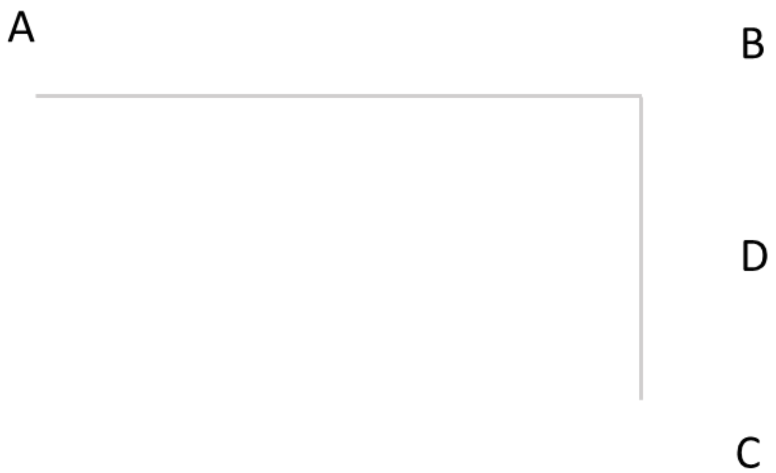
ii) BENDING MOMENT



Maximum moment along AB:

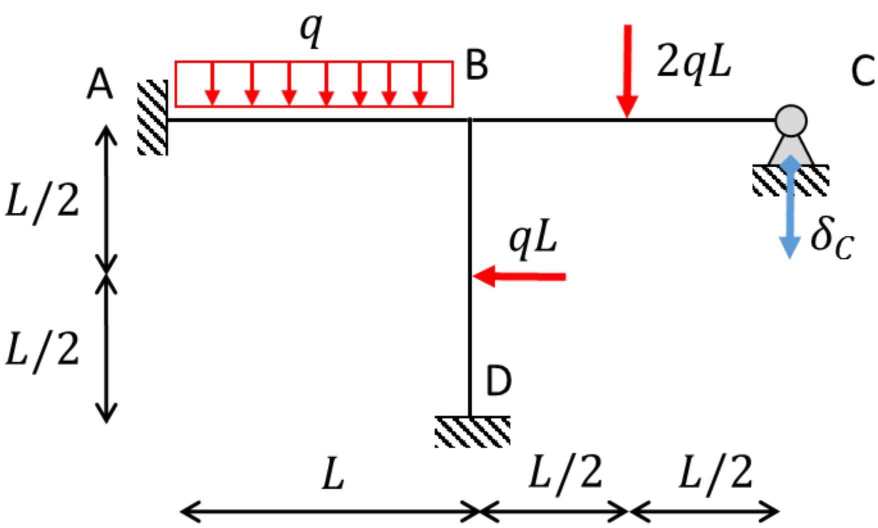
Moment in D:

iii) NORMAL FORCE



2e. Draw the **deformation** of the structure.

3. Consider frame A, B, C, D. Part AB is loaded by a **distributed load q**. In the mid-span of part BC, a **concentrated load 2qL** is applied. In the mid-span of part BD, a **concentrated load qL** is applied. Point C is characterized by a settlement $\delta_C = \frac{47ql^4}{18EI}$. The stiffness EI is constant for all the frame. The change in length of the beams due to normal forces is neglected. **Use the DEFORMATION METHOD to determine the unknown rotations/displacements and the internal and external moments of the structure**, following the guidelines below. The coefficient tables are attached at the end of the sheets.



3a. Degree of static indeterminacy of the structure:

3b. Number of unknown nodal degrees of freedom (displacements and/or rotations) for the structure:

Which between the force and the deformation method is more convenient to solve this structure?

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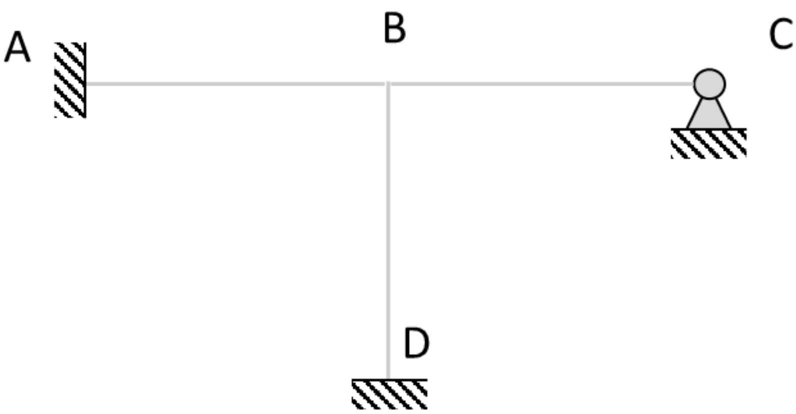
3c. Draw in the Figure below the primary system.



3d. Write the i) **equilibrium condition(s)** and ii) the obtained **value of the unknown displacement(s)/rotation(s)**.

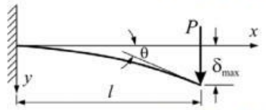
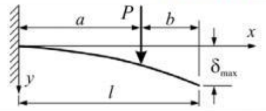
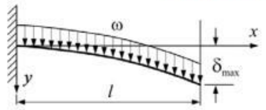
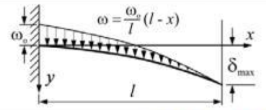
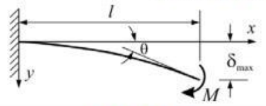
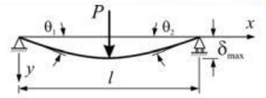
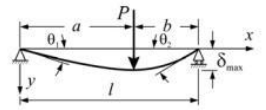
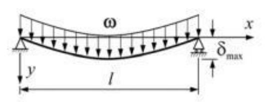
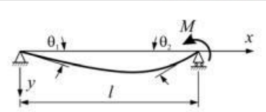
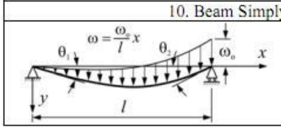
i) Equilibrium condition(s):

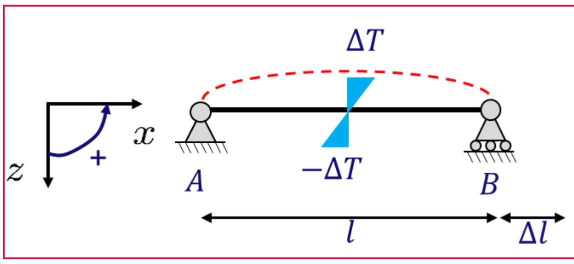
ii) Unknown displacement(s)/rotation(s)

3e. Write the values of the **internal/external moments** in the structure. Indicate also the direction. Hint to verify: check that the sum of moments in point B is equal to zero.

Internal moment in point B (side AB)		*Direction*
Internal moment in point B (side BC)		*Direction*
Internal moment in point B (side BD)		*Direction*
External moment in point A		*Direction*
External moment in point D		*Direction*

COEFFICIENT TABLES FORCE METHOD

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$
BEAM TYPE	**SLOPE AT ENDS**	**DEFLECTION AT ANY SECTION IN TERMS OF x**	**MAXIMUM AND CENTER DEFLECTION**
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$	$\delta_{max} = \frac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}(l^2 - x^2 - b^2)$ for $0 < x < a$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x - a)^3 + (l^2 - b^2)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \frac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{(l^2 - b^2)/3}$ $\delta = \frac{Pb}{48EI}(3l^2 - 4b^2)$ at the center, if $a > b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1 = \theta_2 = \frac{\omega l^3}{24EI}$	$y = \frac{\omega x}{24EI}(l^3 - 2lx^2 + x^3)$	$\delta_{max} = \frac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1 = \frac{Ml}{6EI}$ $\theta_2 = \frac{Ml}{3EI}$	$y = \frac{Mlx}{6EI}\left(1 - \frac{x^2}{l^2}\right)$	$\delta_{max} = \frac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \frac{l}{\sqrt{3}}$ $\delta = \frac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1 = \frac{7\omega_o l^3}{360EI}$ $\theta_2 = \frac{\omega_o l^3}{45EI}$	$y = \frac{\omega_o x}{360lEI}(7l^4 - 10l^2x^2 + 3x^4)$	$\delta_{max} = 0.00652\frac{\omega_o l^4}{EI}$ at $x = 0.519l$ $\delta = 0.00651\frac{\omega_o l^4}{EI}$ at the center



$\Delta T_{top} = \Delta T;\ \Delta T_{bottom} = -\Delta T$

$$\theta_A = -\theta_B = \frac{\alpha \Delta T L}{h}$$

$\Delta l \ = 0$

COEFFICIENT TABLES DEFORMATION METHOD