EINDHOVEN UNIVERSITY OF TECHNOLOGY **DEPARTMENT OF THE BUILT ENVIRONMENT**

Final test: Design of Structures (7P8X0)

Date: 15/08/2018 NAME (in capital letters):

Time: 09.00-12.00 ID.NR.:

Exercises and response-sheets

Please read the following instructions before writing and submitting the answers to the test.

- Put the answers only in the dedicated boxes and/or in the tables and/or in the figures. The answers must be clearly legible. Other comments are not corrected.
- Draw charts and sketches in the pre-printed figures on scale.
- Only one set of response-sheets is given per student. So think carefully before filling in something.
- Hand in the sheets provided with the response-sheets.

Do not remove staples!
The last page (coefficient tables) may be removed from the other pages.

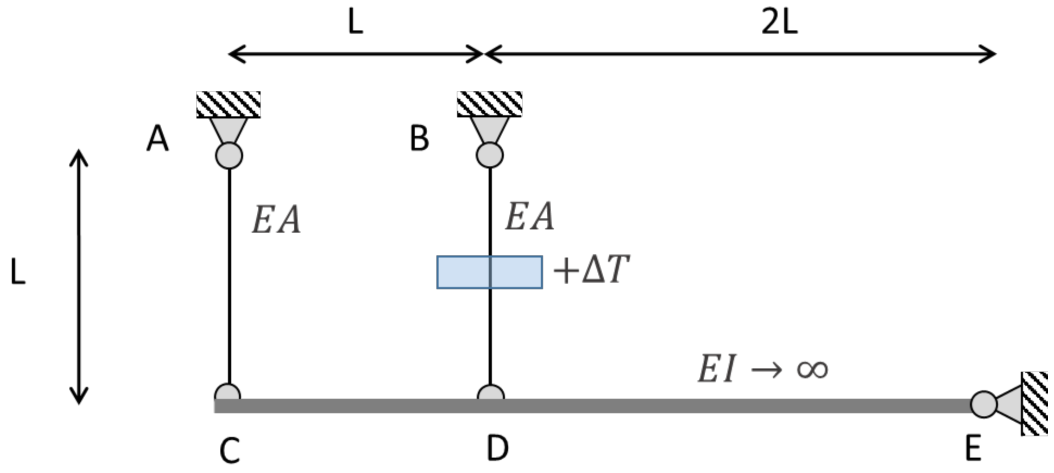
Put the name and ID number of university card at the top of the front page.

Scores for the exercises:
Exercise 1: 14 points
Exercise 2: 18 points
Exercise 3: 18 points

Evaluation of the test: total number of points scored divided by five. The final test counts for 50% of the final grade.

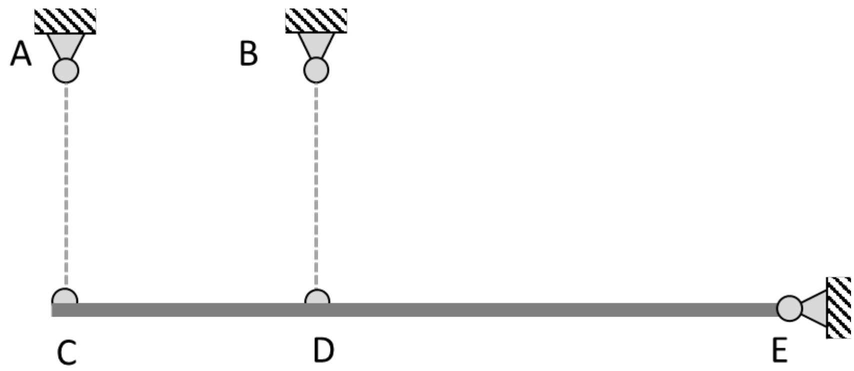
Do not use notebooks, laptop, cell phone etc. A small non-graphic calculator is allowed.

1. A **rigid** horizontal beam (CDE) is supported by two identical vertical rods AC and BD, of length L, and by a hinge in point E. Rod BD is subjected to a temperature variation ΔT. The stiffness EA is constant for all vertical rods; the thermal expansion coefficient is α. Determine the reaction forces in points A, B and E. Solve the exercise by following the guidelines below.

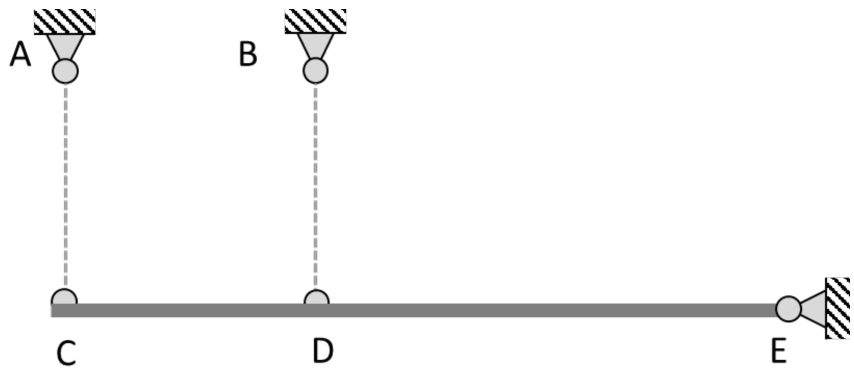


1a. Degree of static indeterminacy of the structure **(1pt):** ☐

1b. Draw in the Figure below the selected statically determinate equivalent system **(1pt).**



1c. Write on the Figure the values and direction of the reaction forces in case corresponding to the action of the unknown redundant force on the equivalent statically determinate structure **(2pt).** Draw the corresponding deformation **(1pt).**



1d. Write the deformation condition **(1 pt)** and elaborate it to obtain the value of the unknown redundant force **(4 pt).**

Deformation condition: Elaboration:

1e. Write the values of the reaction forces in the structure. Indicate also the direction **(3 pt).**

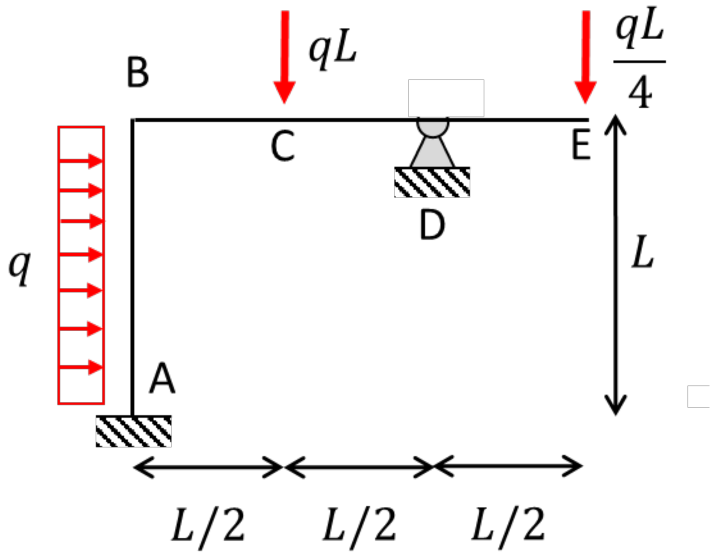
Vertical reaction force in point A		Direction
Vertical reaction force in point B		Direction
Vertical reaction force in point E		Direction

1f. Write the value of the vertical displacement of point D. Indicate also the direction **(1 pt)**

Vertical displacement of point D		Direction

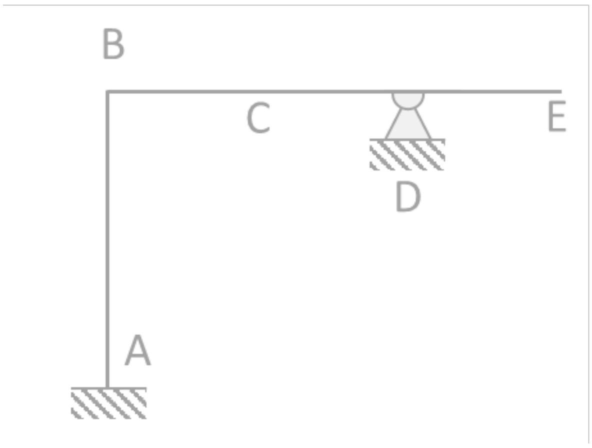
2. Consider frame A, B, C, D, E. Part AB is loaded by a **uniform load q**. In point C, a **point load qL** is applied; in point E, a **point load qL/4** is applied. The stiffness EI is constant for

all the frame. Length change of the beams due to normal forces are neglected. Determine the external forces and draw the free body diagrams using the **force method.** The coefficient tables are attached at the end of the sheets. Solve the exercise by following the guidelines below.



2a. Degree of static indeterminacy of the structure **(1 pt):**

2b. Draw in the Figure below the selected statically determinate equivalent system **(1 pt)**.



2c. Write the deformation condition(s) **(2 pt)** and the obtained value of the unknown redundant forces/moments **(6pt)**.

Deformation conditions:

<u>Elaboration:</u>

2d. Write the values of the reaction forces in the structure. Indicate also the direction **(4 pt)**.

Vertical reaction force in point A		Direction
Horizontal reaction force in point A		Direction
Vertical reaction force in point D		Direction
Horizontal reaction force in point D		Direction

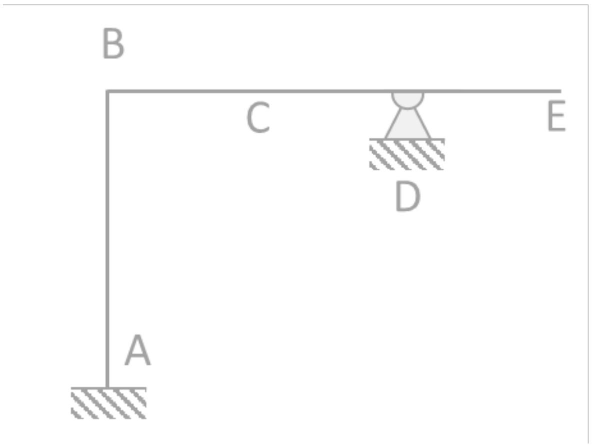
2e. Draw the free body diagrams and the deformed shape for the structure **(4pt).**

Shear

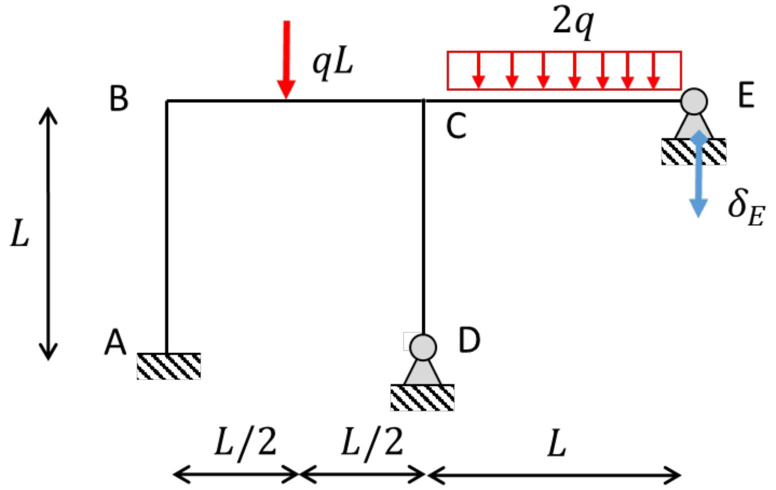
Normal force

Bending moment

Deformed shape



3. Consider frame A, B, C, D, E. Part CE is loaded by a **uniform load 2q**. In part BC, a **point load qL** is applied. Point E is characterized by a settlement $\delta_E = \frac{9ql^4}{16EI}$. The stiffness EI is constant for the entire frame. Length changes of the beams due to normal forces are neglected. Determine the displacements/rotations of points B and C and the external/internal moments using the **deformation method.** The coefficient tables are attached at the end of the sheets.



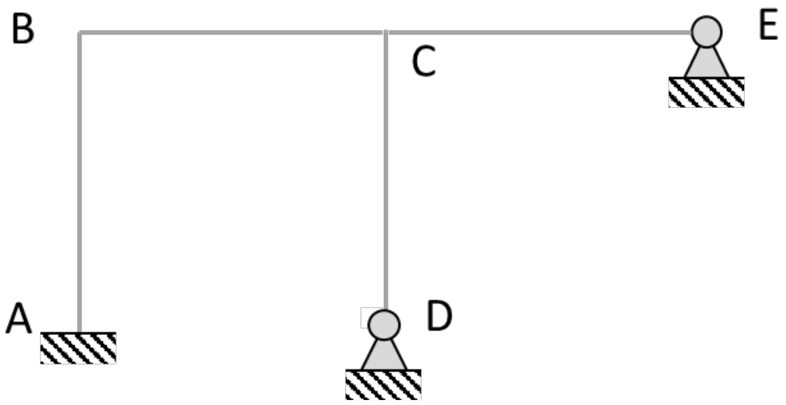
3a. Degree of static indeterminacy of the structure (**1pt**):

3b. Number of unknown nodal degrees of freedom (displacements and/or rotations) for the structure **(2 pt):**

Which between the force and the deformation method is more convenient to solve this structure?

…………..……………………………………………………………………………………….

3c. Draw in the Figure below the primary system **(1 pt).**



3d. Write the equilibrium condition(s) **(2 pt)** and the obtained value of the unknown displacements/rotations **(6 pt)**.

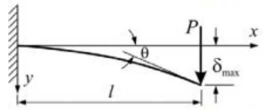
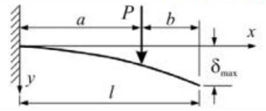
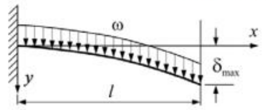
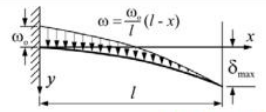
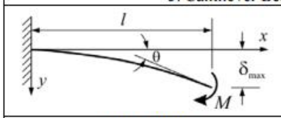
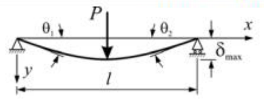
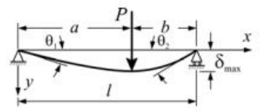
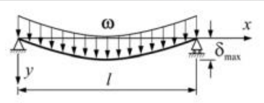
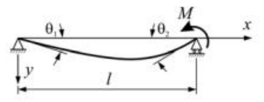
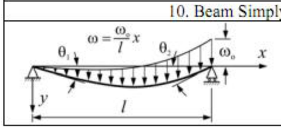
Equilibrium conditions:

Elaboration:

3e. Write the values of the internal/external moments in the structure. Indicate also the direction. Hint: check that the sum of moments in point B and in point C are equal to zero **(6 pt).**

Internal moment in point B (side AB)		Direction
Internal moment in point B (side BC)		Direction
Internal moment in point C (side BC)		Direction
Internal moment in point C (side CD)		Direction
Internal moment in point C (side CE)		Direction
External moment in point A		Direction

COEFFICIENT TABLES FORCE METHOD

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$
BEAM TYPE	**SLOPE AT ENDS**	**DEFLECTION AT ANY SECTION IN TERMS OF x**	**MAXIMUM AND CENTER DEFLECTION**
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$	$\delta_{max} = \frac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}(l^2 - x^2 - b^2)$ for $0 < x < a$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x-a)^3 + (l^2 - b^2)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \frac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{(l^2 - b^2)/3}$ $\delta = \frac{Pb}{48EI}(3l^2 - 4b^2)$ at the center, if $a > b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1 = \theta_2 = \frac{\omega l^3}{24EI}$	$y = \frac{\omega x}{24EI}(l^3 - 2lx^2 + x^3)$	$\delta_{max} = \frac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1 = \frac{Ml}{6EI}$ $\theta_2 = \frac{Ml}{3EI}$	$y = \frac{Mlx}{6EI}\left(1 - \frac{x^2}{l^2}\right)$	$\delta_{max} = \frac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \frac{l}{\sqrt{3}}$ $\delta = \frac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1 = \frac{7\omega_o l^3}{360EI}$ $\theta_2 = \frac{\omega_o l^3}{45EI}$	$y = \frac{\omega_o x}{360lEI}(7l^4 - 10l^2x^2 + 3x^4)$	$\delta_{max} = 0.00652\frac{\omega_o l^4}{EI}$ at $x = 0.519l$ $\delta = 0.00651\frac{\omega_o l^4}{EI}$ at the center

COEFFICIENT TABLES DEFORMATION METHOD