

## Exam Mastering Physics (3NAB0)

Date: January 20, 2016

Time: 09:00 – 12.00

This exam contains ten problems (4 points each). All problems should be worked out on paper. It is mandatory to copy your final answers with units into the table below.

During the exam you are allowed to use (1) the printed formula pages from OASE; (2) (graphical) calculator; (3) dictionary.

Constants:  $g = 9.80 \text{ m/s}^2$ ;

Name: .....

Student ID: .....

Department: .....

Problem	Solution with units	Problem	Solution with units
1a	$k =$	1b	$v =$
2	$v_2 =$		
3a	$T =$	3b	$a =$
4	$f =$		
5	$T =$		
6	$\Delta T =$		
7	$Q$		
8	$d =$		
9a		9b	$f =$
10	$P =$		

### 1) The Spring-Loaded Popgun

The launching mechanism of a toy gun consists of a spring of unknown spring constant (Figure 1a). When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing.

- Determine the spring constant  $k$
- Find the speed  $v$  of the projectile as it moves through the equilibrium position of the spring (where  $x_B = 0.120$  m) as shown in Figure 1b.

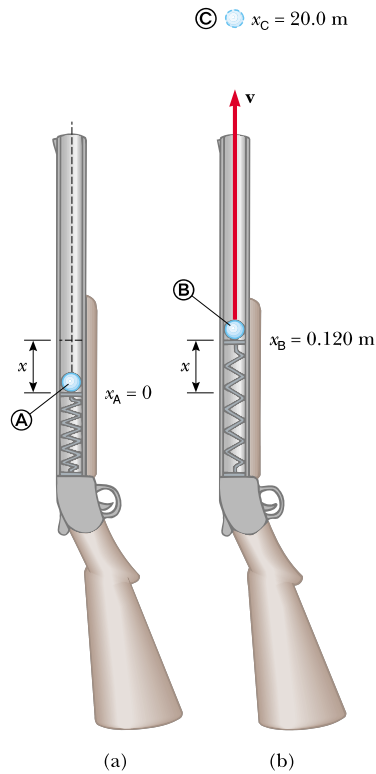
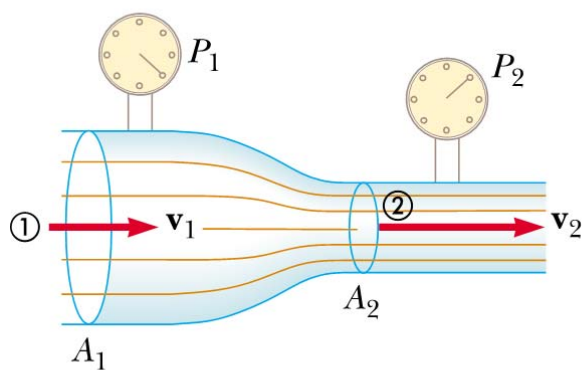


Figure 1.

## 2) The Venturi tube (solve this exercise symbolically)

The horizontal constricted pipe illustrated in Figure 2, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 if the pressure difference  $P_1 - P_2$  is known.



(a)



(b)

Figure 2.

## 3) Rescuing the Hiker

An 80.0-kg hiker is trapped on a mountain ledge following a storm. A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.00 kg, and its length is 15.0 m. A chair of mass 70.0 kg is attached to the end of the cable. The hiker attaches himself to the chair, and the helicopter then accelerates upward. Terrified by hanging from the cable in midair, the hiker tries to signal the pilot by sending transverse pulses up the cable. A pulse takes 0.250 s to travel the length of the cable.

- What is the tension  $T$  in the cable?
- What is the acceleration  $a$  of the helicopter (the mass of the cable can be neglected)?

#### 4) Two speakers driven by the same source

A pair of speakers placed 3.00 m apart are driven by the same oscillator (Figure 3). A listener is originally at point  $O$ , which is located 8.00 m from the center of the line connecting the two speakers. The listener then walks to point  $P$ , which is a perpendicular distance is 0.350 m from  $O$ , before reaching the first minimum in sound intensity. What is the frequency  $f$  of the oscillator? The speed of sound in air 343m/s.

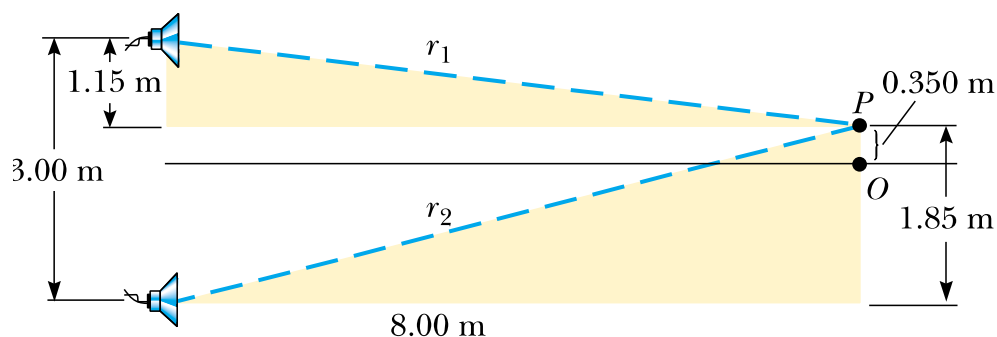


Figure 3

#### 5) The Thermal Electrical Short

An electronic device has been poorly designed so that two bolts attached to different parts of the device almost touch each other in its interior, as in Figure 4. The steel and brass bolts are at different electric potentials and if they touch, a short circuit will develop, damaging the device. If the initial gap between the ends of the bolts is  $5.0 \mu\text{m}$  at  $27^\circ\text{C}$ , at what temperature  $T$  will the bolts touch?

$$\alpha_{\text{Steel}} = 1.1 \times 10^{-5} [\text{K}^{-1}],$$

$$\alpha_{\text{Brass}} = 1.9 \times 10^{-5} [\text{K}^{-1}]$$

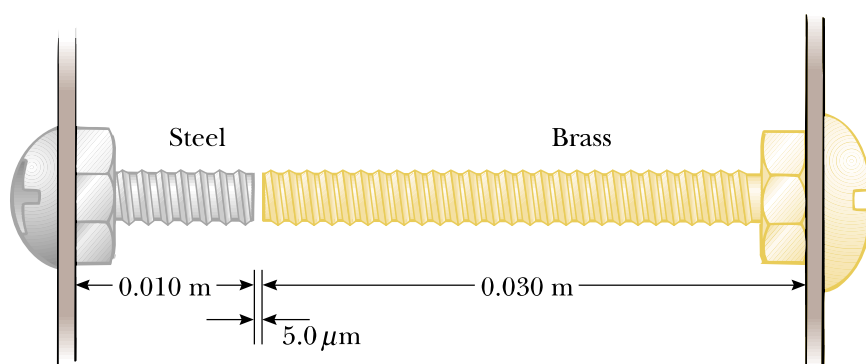


Figure 4.

**6) Fun Time for a Cowboy**

A cowboy fires a silver bullet with a muzzle speed of 200 m/s into the pine wall of a saloon. Assume that all the internal energy generated by the impact remains with the bullet. What is the temperature change  $\Delta T$  of the bullet? The specific heat of silver is 234 J/Kg\*K.

**7) Who Turned Down the Thermostat**

A student is trying to decide what to wear. The surroundings (his bedroom) are at 20.0°C. If the skin temperature of the unclothed student is 35°C, what is the net energy loss  $Q$  from his body in 10.0 min by radiation? Assume that the emissivity of skin is 0.900 and that the surface area of the student is 1.50 m<sup>2</sup>.

**8) The End of the Ski jump**

A ski-jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s, as shown in Figure 5. The landing incline below him falls off with a slope of 35.0°. Where does he land on the incline? Determine the distance  $d$  (see Figure 5).

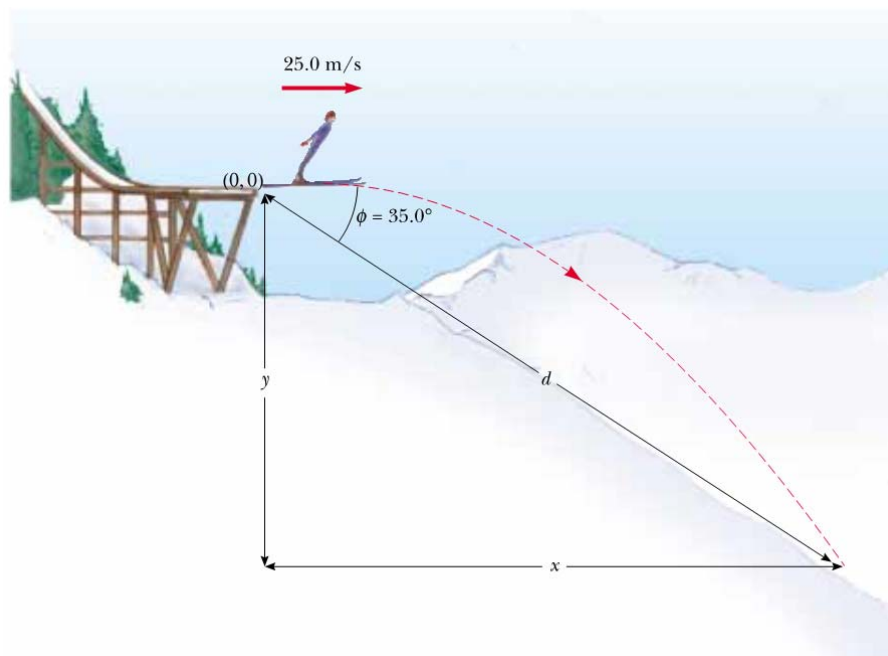


Figure 5.

9)

The person in Figure 6 weighs 800 N. As seen from the front, each light crutch makes an angle of  $22.0^\circ$  with the vertical. Half of the person's weight is supported by the crutches. The other half is supported by the vertical forces of the ground on his feet. Assuming the person is moving with constant velocity and the force exerted by the ground on the crutches acts along the crutches.

- a) Draw the free body diagram of the crutch number 1 (as seen from the front).
- b) Determine the smallest possible coefficient of friction  $f$  between crutches and ground.

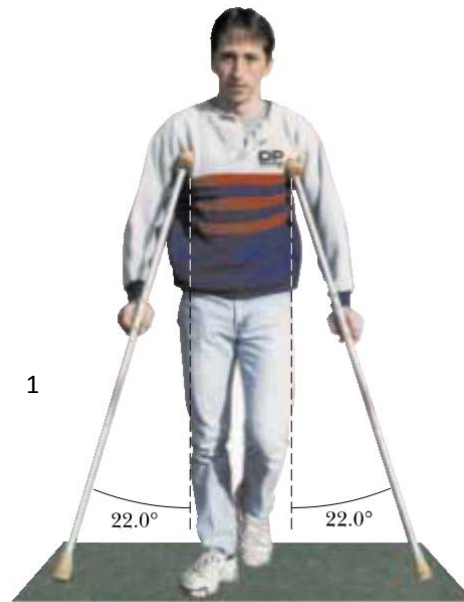


Figure 6.

**10) Car Accelerating Up a Hill (Solve this exercise symbolically)**

Consider a car of mass  $m$  that is accelerating up a hill with a constant slope angle  $\theta$ . An automotive engineer measures the magnitude of the total resistive force to be

$$F_t = (a + b \cdot v^2) \text{ N}$$

where  $v$  is the speed in meters per second. Determine the power  $P$  the engine must deliver to the wheels as a function of speed.

## Som 1

The launching mechanism of a toy gun consists of a spring of unknown spring constant (Fig. 8.9a). When the spring is

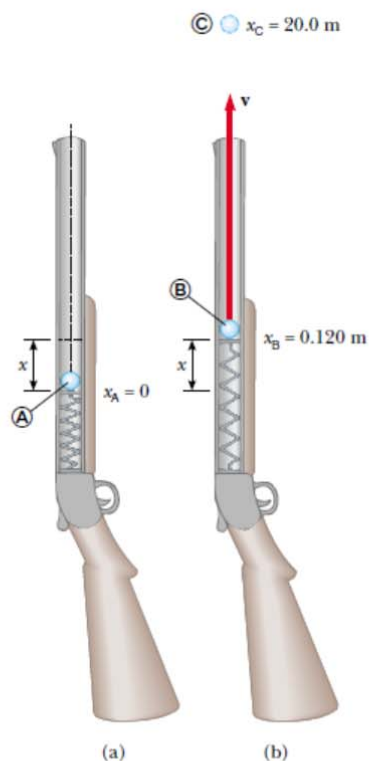


Figure 8.9 (Example 8.5) A spring-loaded popgun.

(B) Find the speed of the projectile as it moves through the equilibrium position of the spring (where  $x_B = 0.120 \text{ m}$ ) as shown in Figure 8.9b.

**Solution** As already noted, the only mechanical energy in the system at (A) is the elastic potential energy  $\frac{1}{2}kx^2$ . The total energy of the system as the projectile moves through the equilibrium position of the spring includes the kinetic energy of the projectile  $\frac{1}{2}mv_B^2$  and the gravitational potential energy  $mgx_B$  of the system. Hence, the principle of conservation of mechanical energy in this case gives

$$E_B = E_A$$

compressed  $0.120 \text{ m}$ , the gun, when fired vertically, is able to launch a  $35.0\text{-g}$  projectile to a maximum height of  $20.0 \text{ m}$  above the position of the projectile before firing.

(A) Neglecting all resistive forces, determine the spring constant.

**Solution** Because the projectile starts from rest, its initial kinetic energy is zero. If we take the zero configuration for the gravitational potential energy of the projectile–spring–Earth system to be when the projectile is at the lowest position  $x_A$ , then the initial gravitational potential energy of the system also is zero. The mechanical energy of this system is conserved because the system is isolated.

Initially, the only mechanical energy in the system is the elastic potential energy stored in the spring of the gun,  $U_{sA} = \frac{1}{2}kx^2$ , where the compression of the spring is  $x = 0.120 \text{ m}$ . The projectile rises to a maximum height  $x_C = h = 20.0 \text{ m}$ , and so the final gravitational potential energy of the system when the projectile reaches its peak is  $mgh$ . The final kinetic energy of the projectile is zero, and the final elastic potential energy stored in the spring is zero. Because the mechanical energy of the system is conserved, we find that

$$\begin{aligned} E_C &= E_A \\ K_C + U_{gC} + U_{sC} &= K_A + U_{gA} + U_{sA} \\ 0 + mgh + 0 &= 0 + 0 + \frac{1}{2}kx^2 \\ k &= \frac{2mgh}{x^2} = \frac{2(0.0350 \text{ kg})(9.80 \text{ m/s}^2)(20.0 \text{ m})}{(0.120 \text{ m})^2} \\ &= 953 \text{ N/m} \end{aligned}$$

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

$$\frac{1}{2}mv_B^2 + mgx_B + 0 = 0 + 0 + \frac{1}{2}kx^2$$

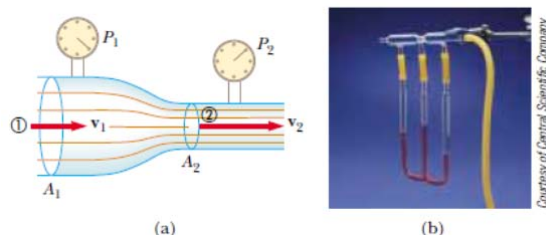
Solving for  $v_B$  gives

$$\begin{aligned} v_B &= \sqrt{\frac{kx^2}{m} - 2gx_B} \\ &= \sqrt{\frac{(953 \text{ N/m})(0.120 \text{ m})^2}{(0.0350 \text{ kg})} - 2(9.80 \text{ m/s}^2)(0.120 \text{ m})} \\ &= 19.7 \text{ m/s} \end{aligned}$$



## Som 2

The horizontal constricted pipe illustrated in Figure 14.20, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 if the pressure difference  $P_1 - P_2$  is known.



**Figure 14.20** (Example 14.9) (a) Pressure  $P_1$  is greater than pressure  $P_2$  because  $v_1 < v_2$ . This device can be used to measure the speed of fluid flow. (b) A Venturi tube, located at the top of the photograph. The higher level of fluid in the middle column shows that the pressure at the top of the column, which is in the constricted region of the Venturi tube, is lower.

**Solution** Because the pipe is horizontal,  $y_1 = y_2$ , and applying Equation 14.8 to points 1 and 2 gives

$$(1) \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

From the equation of continuity,  $A_1 v_1 = A_2 v_2$ , we find that

$$(2) \quad v_1 = \frac{A_2}{A_1} v_2$$

Substituting this expression into Equation (1) gives

$$P_1 + \frac{1}{2}\rho \left( \frac{A_2}{A_1} v_2 \right)^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

We can use this result and the continuity equation to obtain an expression for  $v_1$ . Because  $A_2 < A_1$ , Equation (2) shows us that  $v_2 > v_1$ . This result, together with Equation (1), indicates that  $P_1 > P_2$ . In other words, the pressure is reduced in the constricted part of the pipe.

## Som 3

An 80.0-kg hiker is trapped on a mountain ledge following a storm. A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.00 kg, and its length is 15.0 m. A chair of mass 70.0 kg is attached to the end of the cable. The hiker attaches himself to the chair, and the helicopter then accelerates upward. Terrified by hanging from the cable in midair, the hiker tries to signal the pilot by sending transverse pulses up the cable. A pulse takes 0.250 s to travel the length of the cable. What is the acceleration of the helicopter?

**Solution** To conceptualize this problem, imagine the effect of the acceleration of the helicopter on the cable. The higher the upward acceleration, the larger is the tension in the cable. In turn, the larger the tension, the higher is the speed of pulses on the cable. Thus, we categorize this problem as a combination of one involving Newton's laws and one involving the speed of pulses on a string. To analyze the problem, we use the time interval for the pulse to travel from the hiker to the helicopter to find the speed of the pulses on the cable:

$$v = \frac{\Delta x}{\Delta t} = \frac{15.0 \text{ m}}{0.250 \text{ s}} = 60.0 \text{ m/s}$$

The speed of pulses on the cable is given by Equation 16.18, which allows us to find the tension in the cable:

$$v = \sqrt{\frac{T}{\mu}} \longrightarrow T = \mu v^2 = \left( \frac{8.00 \text{ kg}}{15.0 \text{ m}} \right) (60.0 \text{ m/s})^2$$

$$T = 1.92 \times 10^3 \text{ N}$$

Newton's second law relates the tension in the cable to the acceleration of the hiker and the chair, which is the same as the acceleration of the helicopter:

$$\sum F = ma \longrightarrow T - mg = ma$$

$$a = \frac{T}{m} - g = \frac{1.92 \times 10^3 \text{ N}}{150.0 \text{ kg}} - 9.80 \text{ m/s}^2$$

$$= 3.00 \text{ m/s}^2$$

To finalize this problem, note that a real cable has stiffness in addition to tension. Stiffness tends to return a wire to its original straight-line shape even when it is not under tension. For example, a piano wire straightens if released from a curved shape; package wrapping string does not.

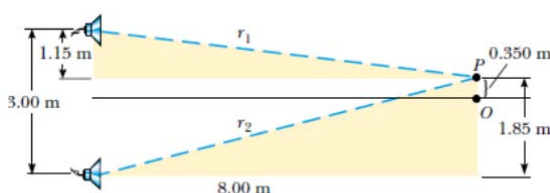
Stiffness represents a restoring force in addition to tension, and increases the wave speed. Consequently, for a real cable, the speed of 60.0 m/s that we determined is most likely associated with a tension lower than  $1.92 \times 10^3 \text{ N}$  and a correspondingly smaller acceleration of the helicopter.



# Som 4

A pair of speakers placed 3.00 m apart are driven by the same oscillator (Fig. 18.6). A listener is originally at point  $O$ , which is located 8.00 m from the center of the line connecting the two speakers. The listener then walks to point  $P$ , which is a perpendicular distance 0.350 m from  $O$ , before reaching the *first minimum* in sound intensity. What is the frequency of the oscillator?

**Solution** To find the frequency, we must know the wavelength of the sound coming from the speakers. With this information, combined with our knowledge of the speed of sound, we can calculate the frequency. The wavelength can be determined from the interference information given. The first minimum occurs when the two waves reaching the listener at point  $P$  are  $180^\circ$  out of phase—in other words, when their path difference  $\Delta r$  equals  $\lambda/2$ . To calculate the path difference, we must first find the path lengths  $r_1$  and  $r_2$ .



**Figure 18.6** (Example 18.1) Two speakers emit sound waves to a listener at  $P$ .

Figure 18.6 shows the physical arrangement of the speakers, along with two shaded right triangles that can be drawn on the basis of the lengths described in the problem. From these triangles, we find that the path lengths are

$$r_1 = \sqrt{(8.00 \text{ m})^2 + (1.15 \text{ m})^2} = 8.08 \text{ m}$$

and

$$r_2 = \sqrt{(8.00 \text{ m})^2 + (1.85 \text{ m})^2} = 8.21 \text{ m}$$

Hence, the path difference is  $r_2 - r_1 = 0.13 \text{ m}$ . Because we require that this path difference be equal to  $\lambda/2$  for the first minimum, we find that  $\lambda = 0.26 \text{ m}$ .

To obtain the oscillator frequency, we use Equation 16.12,  $v = \lambda f$ , where  $v$  is the speed of sound in air, 343 m/s:

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.26 \text{ m}} = 1.3 \text{ kHz}$$

**What If?** What if the speakers were connected out of phase? What happens at point  $P$  in Figure 18.6?

**Answer** In this situation, the path difference of  $\lambda/2$  combines with a phase difference of  $\lambda/2$  due to the incorrect wiring to give a full phase difference of  $\lambda$ . As a result, the waves are in phase and there is a *maximum* intensity at point  $P$ .

# Som 5

An electronic device has been poorly designed so that two bolts attached to different parts of the device almost touch each other in its interior, as in Figure 19.10. The steel and brass bolts are at different electric potentials and if they touch, a short circuit will develop, damaging the device. (We will study electric potential in Chapter 25.) If the initial gap between the ends of the bolts is  $5.0 \mu\text{m}$  at  $27^\circ\text{C}$ , at what temperature will the bolts touch?

**Solution** We can conceptualize the situation by imagining that the ends of both bolts expand into the gap between them as the temperature rises. We categorize this as a thermal expansion problem, in which the *sum* of the changes in length of the two bolts must equal the length of the initial gap between the ends. To analyze the problem, we write this condition mathematically:

$$\Delta L_{\text{br}} + \Delta L_{\text{st}} = \alpha_{\text{br}} L_{i,\text{br}} \Delta T + \alpha_{\text{st}} L_{i,\text{st}} \Delta T = 5.0 \times 10^{-6} \text{ m}$$

Solving for  $\Delta T$ , we find

$$\begin{aligned} \Delta T &= \frac{5.0 \times 10^{-6} \text{ m}}{\alpha_{\text{br}} L_{i,\text{br}} + \alpha_{\text{st}} L_{i,\text{st}}} \\ &= \frac{5.0 \times 10^{-6} \text{ m}}{(19 \times 10^{-6} \text{ }^\circ\text{C}^{-1})(0.030 \text{ m}) + (11 \times 10^{-6} \text{ }^\circ\text{C}^{-1})(0.010 \text{ m})} \\ &= 7.4^\circ\text{C} \end{aligned}$$

Thus, the temperature at which the bolts touch is  $27^\circ\text{C} + 7.4^\circ\text{C} = 34^\circ\text{C}$ . To finalize this problem, note that this temperature is possible if the air conditioning in the building housing the device fails for a long period on a very hot summer day.

# Som 6

A cowboy fires a silver bullet with a muzzle speed of 200 m/s into the pine wall of a saloon. Assume that all the internal energy generated by the impact remains with the bullet. What is the temperature change of the bullet?

**Solution** The kinetic energy of the bullet is

$$K = \frac{1}{2} mv^2$$

Because nothing in the environment is hotter than the bullet, the bullet gains no energy by heat. Its temperature increases because the kinetic energy is transformed to extra

internal energy when the bullet is stopped by the wall. The temperature change is the same as that which would take place if energy  $Q = K$  were transferred by heat from a stove to the bullet. If we imagine this latter process taking place, we can calculate  $\Delta T$  from Equation 20.4. Using  $234 \text{ J/kg} \cdot ^\circ\text{C}$  as the specific heat of silver (see Table 20.1), we obtain

$$(1) \quad \Delta T = \frac{Q}{mc} = \frac{K}{mc} = \frac{\frac{1}{2} m(200 \text{ m/s})^2}{m(234 \text{ J/kg} \cdot ^\circ\text{C})} = 85.5^\circ\text{C}$$

Note that the result does not depend on the mass of the bullet.

## Som 7

A student is trying to decide what to wear. The surroundings (his bedroom) are at  $20.0^\circ\text{C}$ . If the skin temperature of the unclothed student is  $35^\circ\text{C}$ , what is the net energy loss from his body in 10.0 min by radiation? Assume that the emissivity of skin is 0.900 and that the surface area of the student is  $1.50\text{ m}^2$ .

**Solution** Using Equation 20.19, we find that the net rate of energy loss from the skin is

$$\mathcal{P}_{\text{net}} = \sigma A e (T^4 - T_0^4)$$

$$= (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.50 \text{ m}^2) \times (0.900)[(308 \text{ K})^4 - (293 \text{ K})^4] = 125 \text{ W}$$

At this rate, the total energy lost by the skin in 10 min is

$$Q = \mathcal{P}_{\text{net}} \Delta t = (125 \text{ W})(600 \text{ s}) = 7.5 \times 10^4 \text{ J}$$

Note that the energy radiated by the student is roughly equivalent to that produced by two 60-W light bulbs!

## Som 8

A ski-jumper leaves the ski track moving in the horizontal direction with a speed of  $25.0\text{ m/s}$ , as shown in Figure 4.16. The landing incline below him falls off with a slope of  $35.0^\circ$ . Where does he land on the incline?

**Solution** We can *conceptualize* this problem based on observations of winter Olympic ski competitions. We observe the skier to be airborne for perhaps 4 s and go a distance of about 100 m horizontally. We should expect the value of  $d$ , the distance traveled along the incline, to be of the same order of magnitude. We *categorize* the problem as that of a particle in projectile motion.

To *analyze* the problem, it is convenient to select the beginning of the jump as the origin. Because  $v_{xi} = 25.0\text{ m/s}$  and  $v_{yi} = 0$ , the  $x$  and  $y$  component forms of Equation 4.9a are

$$(1) \quad x_f = v_{xi}t = (25.0 \text{ m/s})t$$

$$(2) \quad y_f = v_{yi}t + \frac{1}{2}a_yt^2 = -\frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

From the right triangle in Figure 4.16, we see that the jumper's  $x$  and  $y$  coordinates at the landing point are  $x_f = d \cos 35.0^\circ$  and  $y_f = -d \sin 35.0^\circ$ . Substituting these relationships into (1) and (2), we obtain

$$(3) \quad d \cos 35.0^\circ = (25.0 \text{ m/s})t$$

$$(4) \quad -d \sin 35.0^\circ = -\frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving (3) for  $t$  and substituting the result into (4), we find that  $d = 109\text{ m}$ . Hence, the  $x$  and  $y$  coordinates of the point at which the skier lands are

$$x_f = d \cos 35.0^\circ = (109 \text{ m})\cos 35.0^\circ = 89.3 \text{ m}$$

$$y_f = -d \sin 35.0^\circ = -(109 \text{ m})\sin 35.0^\circ = -62.5 \text{ m}$$

To *finalize* the problem, let us compare these results to our expectations. We expected the horizontal distance to be on the order of 100 m, and our result of 89.3 m is indeed on

Som 9 ; Exchange 170 lb by 80 kg

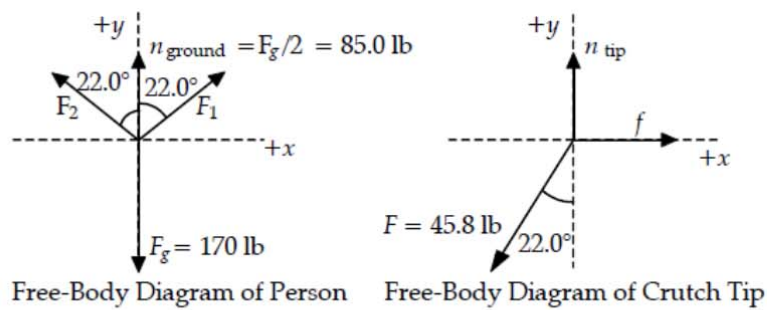


FIG. P5.35

From the free-body diagram of the person,

$$\sum F_x = F_1 \sin(22.0^\circ) - F_2 \sin(22.0^\circ) = 0,$$

which gives

$$F_1 = F_2 = F.$$

Then,  $\sum F_y = 2F \cos 22.0^\circ + 85.0 \text{ lbs} - 170 \text{ lbs} = 0$  yields  $F = 45.8 \text{ lb}$ .

(a) Now consider the free-body diagram of a crutch tip.

$$\sum F_x = f - (45.8 \text{ lb}) \sin 22.0^\circ = 0,$$

or

$$f = 17.2 \text{ lb}.$$

$$\sum F_y = n_{\text{tip}} - (45.8 \text{ lb}) \cos 22.0^\circ = 0,$$

which gives

$$n_{\text{tip}} = 42.5 \text{ lb}.$$

For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so

$$f = (f_s)_{\text{max}} = \mu_s n_{\text{tip}} \text{ and } \mu_s = \frac{f}{n_{\text{tip}}} = \frac{17.2 \text{ lb}}{42.5 \text{ lb}} = \boxed{0.404}.$$

Som 10: take  $a=0$

Consider a car of mass  $m$  that is accelerating up a hill, as shown in Figure 7.20. An automotive engineer measures the magnitude of the total resistive force to be

$$f_t = (218 + 0.70v^2) \text{ N}$$

where  $v$  is the speed in meters per second. Determine the power the engine must deliver to the wheels as a function of speed.

**Solution** The forces on the car are shown in Figure 7.20, in which  $\mathbf{F}$  is the force of friction from the road that propels the car; the remaining forces have their usual meaning.

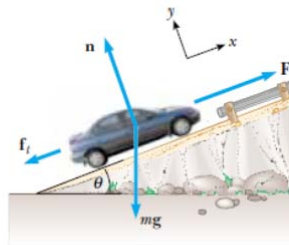


Figure 7.20 (Example 7.15) A car climbs a hill.

$$\begin{aligned} mvg \sin \theta &= (1\,450 \text{ kg})(27 \text{ m/s})(9.80 \text{ m/s}^2)(\sin 10^\circ) \\ &= 67 \text{ kW} = 89 \text{ hp} \end{aligned}$$

$$218v = 218(27 \text{ m/s}) = 5.9 \text{ kW} = 7.9 \text{ hp}$$

$$0.70v^3 = 0.70(27 \text{ m/s})^3 = 14 \text{ kW} = 18 \text{ hp}$$

Hence, the total power required is 126 kW or 167 hp.

Applying Newton's second law to the motion along the road surface, we find that

$$\sum F_x = F - f_t - mg \sin \theta = ma$$

$$\begin{aligned} F &= ma + mg \sin \theta + f_t \\ &= ma + mg \sin \theta + (218 + 0.70v^2) \end{aligned}$$

Therefore, the power required to move the car forward is

$$\mathcal{P} = Fv = mva + mvg \sin \theta + 218v + 0.70v^3$$

The term  $mva$  represents the power that the engine must deliver to accelerate the car. If the car moves at constant speed, this term is zero and the total power requirement is reduced. The term  $mvg \sin \theta$  is the power required to provide a force to balance a component of the gravitational force as the car moves up the incline. This term would be zero for motion on a horizontal surface. The term  $218v$  is the power required to provide a force to balance rolling friction, and the term  $0.70v^3$  is the power needed against air drag.

If we take  $m = 1\,450 \text{ kg}$ ,  $v = 27 \text{ m/s}$  ( $= 60 \text{ mi/h}$ ),  $a = 1.0 \text{ m/s}^2$ , and  $\theta = 10^\circ$ , then the various terms in  $\mathcal{P}$  are calculated to be

$$\begin{aligned} mva &= (1\,450 \text{ kg})(27 \text{ m/s})(1.0 \text{ m/s}^2) \\ &= 39 \text{ kW} = 52 \text{ hp} \end{aligned}$$

Note that the power requirements for traveling at constant speed on a horizontal surface are only 20 kW, or 27 hp (the sum of the last two terms). Furthermore, if the mass were halved (as in the case of a compact car), then the power required also is reduced by almost the same factor.