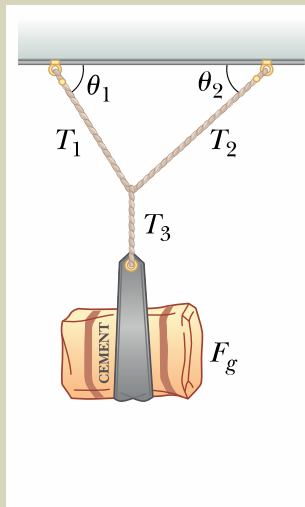


E1 Solve this exercise symbolically.

A bag of cement weighing F_g N hangs in equilibrium from three wires as shown in Figure 1. Two of the wires make angles θ_1 and θ_2 . Find the tensions T_1 and T_2 , using θ_1 , θ_2 and F_g .



in three wires.

25. A bag of cement whose weight is F_g hangs in equilibrium from three wires as shown in Figure P5.25. Two of the wires make angles θ_1 and θ_2 with the horizontal. Assuming the system is in equilibrium, show that the tension in the left-hand wire is

$$T_1 = \frac{F_g \cos \theta_2}{\sin(\theta_1 + \theta_2)}$$

Solution

Conceptualize: We follow the same steps as we would in the numerical problem 24 in the text. F_g , θ_1 , and θ_2 count as known quantities. For the bag of cement to be in equilibrium, the tension T_3 in the vertical wire must be equal to F_g .

Categorize: From Newton's second law for an object with zero acceleration we can write down x and y component equations. The magnitudes of T_1 and T_2 are unknown, but we can take components of these two tensions just as if they were known forces. Then we can eliminate T_2 by substitution and solve for T_1 .

Analyze: We use the particle in equilibrium model. Draw a free-body diagram for the knot where the three ropes are joined. Choose the x axis to be horizontal and apply Newton's second law in component form.

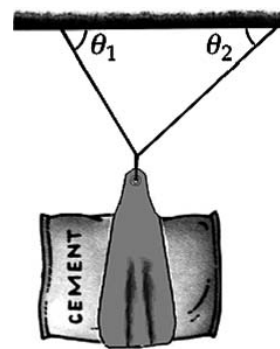
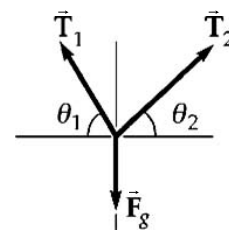


Figure P5.25



$$\sum F_x = 0: \quad T_2 \cos \theta_2 - T_1 \cos \theta_1 = 0 \quad [1]$$

$$\sum F_y = 0: \quad T_2 \sin \theta_2 + T_1 \sin \theta_1 - F_g = 0 \quad [2]$$

Solve equation [1] for $T_2 = \frac{T_1 \cos \theta_1}{\cos \theta_2}$

Substitute this expression for T_2 into equation [2]:

$$\left(\frac{T_1 \cos \theta_1}{\cos \theta_2} \right) \sin \theta_2 + T_1 \sin \theta_1 = F_g$$

Solve for $T_1 = \frac{F_g \cos \theta_2}{\cos \theta_1 \sin \theta_2 + \sin \theta_1 \cos \theta_2}$

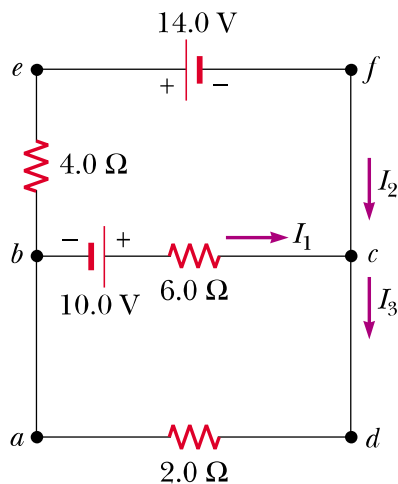
Use the trigonometric identity found in Appendix B.4,

$$\sin(\theta_1 + \theta_2) = \cos \theta_1 \sin \theta_2 + \sin \theta_1 \cos \theta_2$$

to find $T_1 = \frac{F_g \cos \theta_2}{\sin(\theta_1 + \theta_2)}$ ■

Finalize: The equation indicates that the tension is directly proportional to the weight of the bag. As $\sin(\theta_1 + \theta_2)$ approaches zero (as the angle between the two upper ropes approaches 180°) the tension goes to infinity. Making the right-hand rope horizontal maximizes the tension in the left-hand rope, according to the proportionality of T_1 to $\cos \theta_2$. If the right-hand rope is vertical, the tension in the left-hand rope is zero. All this information is contained in the symbolic answer, and not in the numerical answer to problem 24.

E2. A electric circuit

Find the currents I_1 , I_2 , and I_3 in the circuit shown in Figure 2

Categorize We cannot simplify the circuit by the rules associated with combining resistances in series and in parallel. (If the 10.0-V battery were removed and replaced by a wire from b to the 6.0- Ω resistor, we could reduce the remaining circuit.) Because the circuit is not a simple series and parallel combination of resistances, this problem is one in which we must use Kirchhoff's rules.

Analyze We arbitrarily choose the directions of the currents as labeled in Figure 28.15.

Apply Kirchhoff's junction rule to junction c :

We now have one equation with three unknowns: I_1 , I_2 , and I_3 . There are three loops in the circuit: $abcda$, $befcb$, and $aefta$. We need only two loop equations to determine the unknown currents. (The third loop equation would give no new information.) Let's choose to traverse these loops in the clockwise direction. Apply Kirchhoff's loop rule to loops $abcda$ and $befcb$:

Solve Equation (1) for I_3 and substitute into Equation (2):

Multiply each term in Equation (3) by 4 and each term in Equation (4) by 3:

Add Equation (6) to Equation (5) to eliminate I_1 and find I_2 :

Use this value of I_2 in Equation (3) to find I_1 :

Use Equation (1) to find I_3 :

Finalize Because our values for I_2 and I_3 are negative, the directions of these currents are opposite those indicated in Figure 28.15. The numerical values for the currents are correct. Despite the incorrect direction, we *must* continue to use these negative values in subsequent calculations because our equations were established with our original choice of direction. What would have happened had we left the current directions as labeled in Figure 28.15 but traversed the loops in the opposite direction?

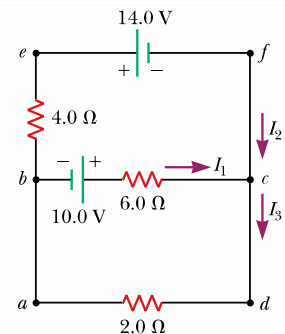


Figure 28.15 (Example 28.7) A circuit containing different branches.

$$(1) \quad I_1 + I_2 - I_3 = 0$$

$$abcda: (2) \quad 10.0 \text{ V} - (6.0 \, \Omega)I_1 - (2.0 \, \Omega)I_3 = 0$$

$$befcb: -(4.0 \, \Omega)I_2 - 14.0 \text{ V} + (6.0 \, \Omega)I_1 - 10.0 \text{ V} = 0$$

$$(3) \quad -24.0 \text{ V} + (6.0 \, \Omega)I_1 - (4.0 \, \Omega)I_2 = 0$$

$$10.0 \text{ V} - (6.0 \, \Omega)I_1 - (2.0 \, \Omega)(I_1 + I_2) = 0$$

$$(4) \quad 10.0 \text{ V} - (8.0 \, \Omega)I_1 - (2.0 \, \Omega)I_2 = 0$$

$$(5) \quad -96.0 \text{ V} + (24.0 \, \Omega)I_1 - (16.0 \, \Omega)I_2 = 0$$

$$(6) \quad 30.0 \text{ V} - (24.0 \, \Omega)I_1 - (6.0 \, \Omega)I_2 = 0$$

$$-66.0 \text{ V} - (22.0 \, \Omega)I_2 = 0$$

$$I_2 = -3.0 \text{ A}$$

$$-24.0 \text{ V} + (6.0 \, \Omega)I_1 - (4.0 \, \Omega)(-3.0 \text{ A}) = 0$$

$$-24.0 \text{ V} + (6.0 \, \Omega)I_1 + 12.0 \text{ V} = 0$$

$$I_1 = 2.0 \text{ A}$$

$$I_3 = I_1 + I_2 = 2.0 \text{ A} - 3.0 \text{ A} = -1.0 \text{ A}$$

E3. A collision.

A 1500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2500-kg truck traveling north at a speed of 20.0 m/s as shown in Figure 3.

Find the direction and magnitude of the velocity of the wreck.

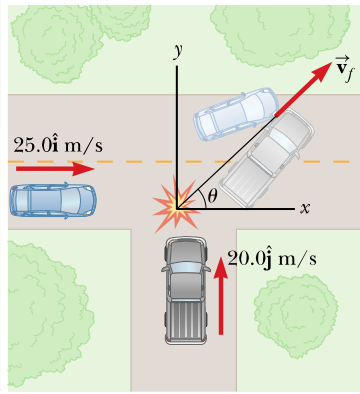


Figure 3.

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg truck traveling north at a speed of 20.0 m/s as shown in Figure 9.12. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.

SOLUTION

Conceptualize Figure 9.12 should help you conceptualize the situation before and after the collision. Let us choose east to be along the positive x direction and north to be along the positive y direction.

Categorize Because we consider moments immediately before and immediately after the collision as defining our time interval, we ignore the small effect that friction would have on the wheels of the vehicles and model the system of two vehicles as isolated in terms of momentum. We also ignore the vehicles' sizes and model them as particles. The collision is perfectly inelastic because the car and the truck stick together after the collision.

Analyze Before the collision, the only object having momentum in the x direction is the car. Therefore, the magnitude of the total initial momentum of the system (car plus truck) in the x direction is that of only the car. Similarly, the total initial momentum of the system in the y direction is that of the truck. After the collision, let us assume the wreckage moves at an angle θ with respect to the x axis with speed v_f .

Equate the initial and final momenta of the system in the x direction:

$$\sum p_{xi} = \sum p_{xf} \rightarrow (1) \quad m_1 v_{1i} = (m_1 + m_2) v_f \cos \theta$$

Equate the initial and final momenta of the system in the y direction:

$$\sum p_{yi} = \sum p_{yf} \rightarrow (2) \quad m_2 v_{2i} = (m_1 + m_2) v_f \sin \theta$$

Divide Equation (2) by Equation (1):

$$\frac{m_2 v_{2i}}{m_1 v_{1i}} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

Solve for θ and substitute numerical values:

$$\theta = \tan^{-1} \left(\frac{m_2 v_{2i}}{m_1 v_{1i}} \right) = \tan^{-1} \left[\frac{(2\,500\text{ kg})(20.0\text{ m/s})}{(1\,500\text{ kg})(25.0\text{ m/s})} \right] = 53.1^\circ$$

Use Equation (2) to find the value of v_f and substitute numerical values:

$$v_f = \frac{m_2 v_{2i}}{(m_1 + m_2) \sin \theta} = \frac{(2\,500\text{ kg})(20.0\text{ m/s})}{(1\,500\text{ kg} + 2\,500\text{ kg}) \sin 53.1^\circ} = 15.6\text{ m/s}$$

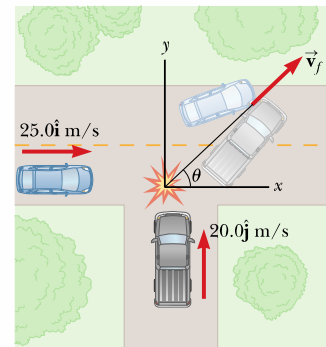


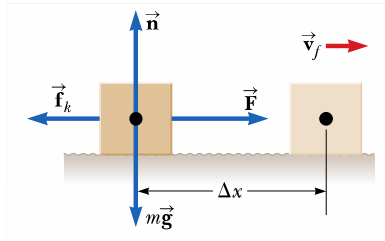
Figure 9.12 (Example 9.8) An eastbound car colliding with a northbound truck.

Finalize Notice that the angle θ is qualitatively in agreement with Figure 9.12. Also notice that the final speed of the combination is less than the initial speeds of the two cars. This result is consistent with the kinetic energy of the system being reduced in an inelastic collision. It might help if you draw the momentum vectors of each vehicle before the collision and the two vehicles together after the collision.

E4. Solve this exercise symbolically.

A block with mass m initially at rest is pulled to the right along a horizontal surface by a constant horizontal force F .

Find the speed of the block after it has moved X meters, if the surfaces in contact have a coefficient of kinetic friction of μ .



A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.

SOLUTION

Conceptualize This example is Example 7.6 (page 178), modified so that the surface is no longer frictionless. The rough surface applies a friction force on the block opposite to the applied force. As a result, we expect the speed to be lower than that found in Example 7.6.

Categorize The block is pulled by a force and the surface is rough, so we model the block-surface system as nonisolated with a nonconservative force acting.

Analyze Active Figure 8.8a illustrates this situation. Neither the normal force nor the gravitational force does work on the system because their points of application are displaced horizontally.

Find the work done on the system by the applied force just as in Example 7.6:

$$\sum W_{\text{other forces}} = W_F = F \Delta x$$

Apply the particle in equilibrium model to the block in the vertical direction:

$$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$

Find the magnitude of the friction force:

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0 \text{ kg})(9.80 \text{ m/s}^2) = 8.82 \text{ N}$$

Find the final speed of the block from Equation 8.14:

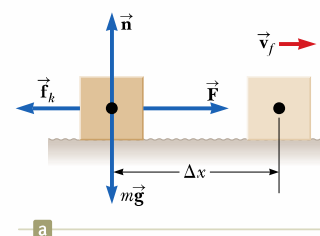
$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - f_k d + W_F$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}(-f_k d + F \Delta x)}$$

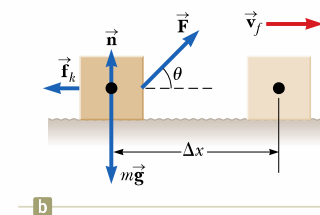
Substitute numerical values:

$$v_f = \sqrt{0 + \frac{2}{6.0 \text{ kg}}[-(8.82 \text{ N})(3.0 \text{ m}) + (12 \text{ N})(3.0 \text{ m})]} = 1.8 \text{ m/s}$$

Finalize As expected, this value is less than the 3.5 m/s found in the case of the block sliding on a frictionless surface (see Example 7.6). The difference in kinetic energies between the block in Example 7.6 and the block in this example is equal to the increase in internal energy of the block-surface system in this example.



a



b

ACTIVE FIGURE 8.8

(Example 8.4) (a) A block pulled to the right on a rough surface by a constant horizontal force. (b) The applied force is at an angle θ to the horizontal.

E5. The Broken Clock Radio

Your clock radio awakens you with a steady and irritating sound of frequency 600 Hz. One morning, it malfunctions and cannot be turned off. In frustration, you drop the clock radio out of your fourth-story dorm window, 15.0 m from the ground. Assume the speed of sound is 343 m/s. As you listen to the falling clock radio, what frequency do you hear just before you hear it striking the ground?

Your clock radio awakens you with a steady and irritating sound of frequency 600 Hz. One morning, it malfunctions and cannot be turned off. In frustration, you drop the clock radio out of your fourth-story dorm window, 15.0 m from the ground. Assume the speed of sound is 343 m/s. As you listen to the falling clock radio, what frequency do you hear just before you hear it striking the ground?

SOLUTION

Conceptualize The speed of the clock radio increases as it falls. Therefore, it is a source of sound moving away from you with an increasing speed so the frequency you hear should be less than 600 Hz.

Categorize We categorize this problem as one in which we combine the particle under constant acceleration model for the falling radio with our understanding of the frequency shift of sound due to the Doppler effect.

Analyze Because the clock radio is modeled as a particle under constant acceleration due to gravity, use Equation 2.13 to express the speed of the source of sound:

$$(1) \quad v_s = v_{yi} + a_y t = 0 - gt = -gt$$

From Equation 2.16, find the time at which the clock radio strikes the ground:

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2 = 0 + 0 - \frac{1}{2}gt^2 \rightarrow t = \sqrt{-\frac{2y_f}{g}}$$

Substitute into Equation (1):

$$v_s = (-g)\sqrt{-\frac{2y_f}{g}} = -\sqrt{-2gy_f}$$

Use Equation 17.19 to determine the Doppler-shifted frequency heard from the falling clock radio:

$$f' = \left[\frac{v + 0}{v - (-\sqrt{-2gy_f})} \right] f = \left(\frac{v}{v + \sqrt{-2gy_f}} \right) f$$

Substitute numerical values:

$$\begin{aligned} f' &= \left[\frac{343 \text{ m/s}}{343 \text{ m/s} + \sqrt{-2(9.80 \text{ m/s}^2)(-15.0 \text{ m})}} \right] (600 \text{ Hz}) \\ &= 571 \text{ Hz} \end{aligned}$$

Finalize The frequency is lower than the actual frequency of 600 Hz because the clock radio is moving away from you. If it were to fall from a higher floor so that it passes below $y = -15.0 \text{ m}$, the clock radio would continue to accelerate and the frequency would continue to drop.

E6. A picnic cooler

A Styrofoam cooler (Figure 4) has total wall area (including the lid) of 0.80 m^2 and wall thickness 2.0 cm . It is filled with ice, water, and cans of Omni-Cola, all at 0°C . What is the rate of heat flow into the cooler if the temperature of the outside wall is 30°C ? How much ice melts in 3 hours?

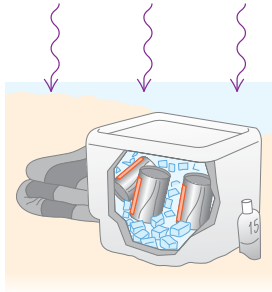


Figure 4.

A Styrofoam cooler (Fig. 17.24a) has total wall area (including the lid) of 0.80 m^2 and wall thickness 2.0 cm . It is filled with ice, water, and cans of Omni-Cola, all at 0°C . What is the rate of heat flow into the cooler if the temperature of the outside wall is 30°C ? How much ice melts in 3 hours?

SOLUTION

IDENTIFY and SET UP: The target variables are the heat current H and the mass m of ice melted. We use Eq. (17.21) to determine H and Eq. (17.20) to determine m .

EXECUTE: We assume that the total heat flow is the same as it would be through a flat Styrofoam slab of area 0.80 m^2 and thickness $2.0 \text{ cm} = 0.020 \text{ m}$ (Fig. 17.24b). We find k from Table 17.5. From Eq. (17.21),

$$H = kA \frac{T_H - T_C}{L} = (0.027 \text{ W/m} \cdot \text{K})(0.80 \text{ m}^2) \frac{30^\circ\text{C} - 0^\circ\text{C}}{0.020 \text{ m}} = 32.4 \text{ W} = 32.4 \text{ J/s}$$

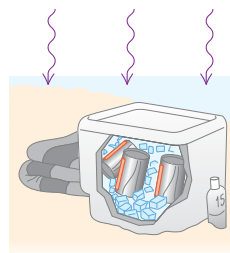
The total heat flow is $Q = Ht$, with $t = 3 \text{ h} = 10,800 \text{ s}$. From Table 17.4, the heat of fusion of ice is $L_f = 3.34 \times 10^5 \text{ J/kg}$, so from Eq. (17.20) the mass of ice that melts is

$$m = \frac{Q}{L_f} = \frac{(32.4 \text{ J/s})(10,800 \text{ s})}{3.34 \times 10^5 \text{ J/kg}} = 1.0 \text{ kg}$$

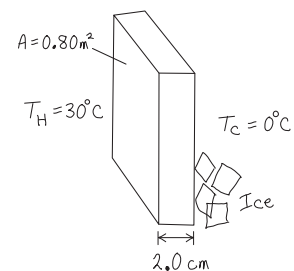
EVALUATE: The low heat current is a result of the low thermal conductivity of Styrofoam.

17.24 Conduction of heat across the walls of a Styrofoam cooler.

(a) A cooler at the beach



(b) Our sketch for this problem



E7. An old car

The shock absorbers in an old car with mass 1000 kg are completely worn out. When a 980 N person climbs slowly into the car at its center of gravity, the car sinks 2.8 cm. The car (with the person aboard) hits a bump, and the car starts oscillating up and down in Simple Harmonic Motion. Model the car and person as a single body on a single spring, and find the period and frequency of the oscillation.

Example 14.6 Vertical SHM in an old car

The shock absorbers in an old car with mass 1000 kg are completely worn out. When a 980-N person climbs slowly into the car at its center of gravity, the car sinks 2.8 cm. The car (with the person aboard) hits a bump, and the car starts oscillating up and down in SHM. Model the car and person as a single body on a single spring, and find the period and frequency of the oscillation.

SOLUTION

IDENTIFY and SET UP: The situation is like that shown in Fig. 14.18. The compression of the spring when the person's weight is added tells us the force constant, which we can use to find the period and frequency (the target variables).

EXECUTE: When the force increases by 980 N, the spring compresses an additional 0.028 m, and the x -coordinate of the car

changes by -0.028 m. Hence the effective force constant (including the effect of the entire suspension) is

$$k = -\frac{F_x}{x} = -\frac{980 \text{ N}}{-0.028 \text{ m}} = 3.5 \times 10^4 \text{ kg/s}^2$$

The person's mass is $w/g = (980 \text{ N})/(9.8 \text{ m/s}^2) = 100 \text{ kg}$. The total oscillating mass is $m = 1000 \text{ kg} + 100 \text{ kg} = 1100 \text{ kg}$. The period T is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1100 \text{ kg}}{3.5 \times 10^4 \text{ kg/s}^2}} = 1.11 \text{ s}$$

The frequency is $f = 1/T = 1/(1.11 \text{ s}) = 0.90 \text{ Hz}$.

EVALUATE: A persistent oscillation with a period of about 1 second makes for a very unpleasant ride. The purpose of shock absorbers is to make such oscillations die out (see Section 14.7).

E8. A solid gold statue

A 15.0-kg solid gold statue is raised from the sea bottom (Figure 5). What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?

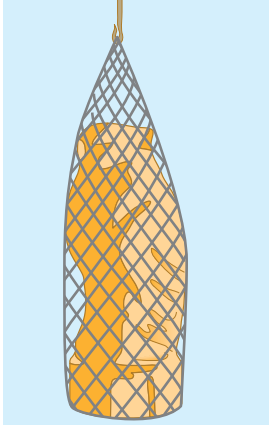


Figure 5

Specific mass of gold 19300 Kg/m^3

A 15.0-kg solid gold statue is raised from the sea bottom (Fig. 12.13a). What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?

SOLUTION

IDENTIFY and SET UP: In both cases the statue is in equilibrium and experiences three forces: its weight, the cable tension, and a buoyant force equal in magnitude to the weight of the fluid displaced by the statue (seawater in part (a), air in part (b)). Figure 12.13b shows the free-body diagram for the statue. Our target variables are the values of the tension in seawater (T_{sw}) and in air (T_{air}). We are given the mass m_{statue} , and we can calculate the buoyant force in seawater (B_{sw}) and in air (B_{air}) using Archimedes's principle.

EXECUTE: (a) To find B_{sw} , we first find the statue's volume V using the density of gold from Table 12.1:

$$V = \frac{m_{\text{statue}}}{\rho_{\text{gold}}} = \frac{15.0 \text{ kg}}{19.3 \times 10^3 \text{ kg/m}^3} = 7.77 \times 10^{-4} \text{ m}^3$$

The buoyant force B_{sw} equals the weight of this same volume of seawater. Using Table 12.1 again:

$$\begin{aligned} B_{\text{sw}} &= w_{\text{sw}} = m_{\text{sw}}g = \rho_{\text{sw}}Vg \\ &= (1.03 \times 10^3 \text{ kg/m}^3)(7.77 \times 10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 7.84 \text{ N} \end{aligned}$$

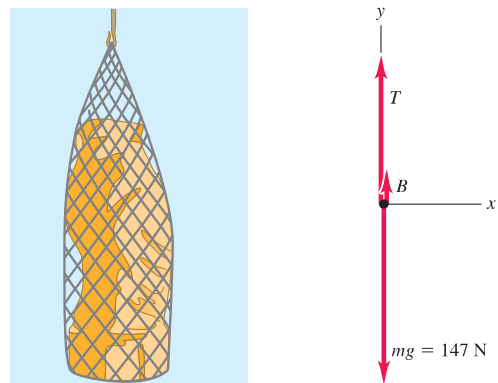
The statue is at rest, so the net external force acting on it is zero. From Fig. 12.13b,

$$\begin{aligned} \sum F_y &= B_{\text{sw}} + T_{\text{sw}} + (-m_{\text{statue}}g) = 0 \\ T_{\text{sw}} &= m_{\text{statue}}g - B_{\text{sw}} = (15.0 \text{ kg})(9.80 \text{ m/s}^2) - 7.84 \text{ N} \\ &= 147 \text{ N} - 7.84 \text{ N} = 139 \text{ N} \end{aligned}$$

the statue. The denser the fluid, the greater the buoyant force and the smaller the cable tension. If the fluid had the same density as the statue, the buoyant force would be equal to the statue's weight and the tension would be zero (the cable would go slack). If the fluid

12.13 What is the tension in the cable hoisting the statue?

(a) Immersed statue in equilibrium (b) Free-body diagram of statue



A spring scale attached to the upper end of the cable will indicate a tension 7.84 N less than the statue's actual weight $m_{\text{statue}}g = 147 \text{ N}$.

(b) The density of air is about 1.2 kg/m^3 , so the buoyant force of air on the statue is

$$\begin{aligned} B_{\text{air}} &= \rho_{\text{air}}Vg = (1.2 \text{ kg/m}^3)(7.77 \times 10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 9.1 \times 10^{-3} \text{ N} \end{aligned}$$

This is negligible compared to the statue's actual weight $m_{\text{statue}}g = 147 \text{ N}$. So within the precision of our data, the tension in the cable with the statue in air is $T_{\text{air}} = m_{\text{statue}}g = 147 \text{ N}$.

EVALUATE: Note that the buoyant force is proportional to the density of the *fluid* in which the statue is immersed, *not* the density of

Continued

were denser than the statue, the tension would be *negative*: The buoyant force would be greater than the statue's weight, and a downward force would be required to keep the statue from rising upward.

E9. A motorcycle stunt

A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff, and velocity 0.50 s after it leaves the edge of the cliff.

A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff, and velocity 0.50 s after it leaves the edge of the cliff.

SOLUTION

IDENTIFY and SET UP: Figure 3.22 shows our sketch of the motorcycle's trajectory. He is in projectile motion as soon as he leaves the edge of the cliff, which we choose to be the origin of coordinates so $x_0 = 0$ and $y_0 = 0$. His initial velocity \vec{v}_0 at the edge of the cliff is horizontal (that is, $\alpha_0 = 0$), so its components are $v_{0x} = v_0 \cos \alpha_0 = 9.0 \text{ m/s}$ and $v_{0y} = v_0 \sin \alpha_0 = 0$. To find the motorcycle's position at $t = 0.50 \text{ s}$, we use Eqs. (3.20) and (3.21); we then find the distance from the origin using Eq. (3.24). Finally, we use Eqs. (3.22) and (3.23) to find the velocity components at $t = 0.50 \text{ s}$.

EXECUTE: From Eqs. (3.20) and (3.21), the motorcycle's x - and y -coordinates at $t = 0.50 \text{ s}$ are

$$x = v_{0x}t = (9.0 \text{ m/s})(0.50 \text{ s}) = 4.5 \text{ m}$$

$$y = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.80 \text{ m/s}^2)(0.50 \text{ s})^2 = -1.2 \text{ m}$$

The negative value of y shows that the motorcycle is below its starting point.

From Eq. (3.24), the motorcycle's distance from the origin at $t = 0.50 \text{ s}$ is

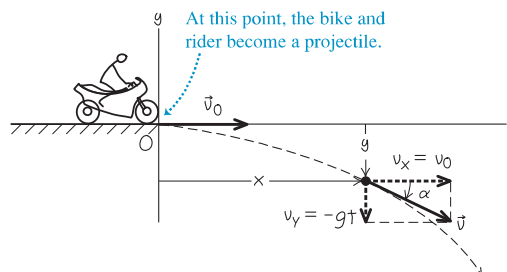
$$r = \sqrt{x^2 + y^2} = \sqrt{(4.5 \text{ m})^2 + (-1.2 \text{ m})^2} = 4.7 \text{ m}$$

From Eqs. (3.22) and (3.23), the velocity components at $t = 0.50 \text{ s}$ are

$$v_x = v_{0x} = 9.0 \text{ m/s}$$

$$v_y = -gt = (-9.80 \text{ m/s}^2)(0.50 \text{ s}) = -4.9 \text{ m/s}$$

3.22 Our sketch for this problem.



The motorcycle has the same horizontal velocity v_x as when it left the cliff at $t = 0$, but in addition there is a downward (negative) vertical velocity v_y . The velocity vector at $t = 0.50 \text{ s}$ is

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = (9.0 \text{ m/s})\hat{i} + (-4.9 \text{ m/s})\hat{j}$$

From Eq. (3.25), the speed (magnitude of the velocity) at $t = 0.50 \text{ s}$ is

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{(9.0 \text{ m/s})^2 + (-4.9 \text{ m/s})^2} = 10.2 \text{ m/s} \end{aligned}$$

From Eq. (3.26), the angle α of the velocity vector is

$$\alpha = \arctan \frac{v_y}{v_x} = \arctan \left(\frac{-4.9 \text{ m/s}}{9.0 \text{ m/s}} \right) = -29^\circ$$

The velocity is 29° below the horizontal.

EVALUATE: Just as in Fig. 3.17, the motorcycle's horizontal motion is unchanged by gravity; the motorcycle continues to move horizontally at 9.0 m/s, covering 4.5 m in 0.50 s. The motorcycle initially has zero vertical velocity, so it falls vertically just like a body released from rest and descends a distance $\frac{1}{2}gt^2 = 1.2 \text{ m}$ in 0.50 s.

E10. An enclosed tank

An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom (Fig. 6). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P . Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.

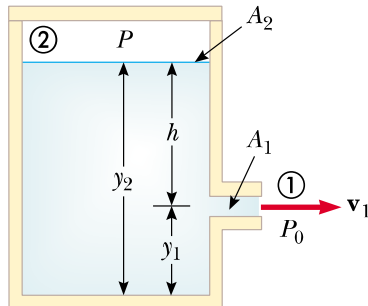


Figure 6.

Example 14.10 Torricelli's Law**Interactive**

An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom (Fig. 14.21). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P . Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.

Solution Because $A_2 \gg A_1$, the liquid is approximately at rest at the top of the tank, where the pressure is P . Applying Bernoulli's equation to points 1 and 2 and noting that at the hole P_1 is equal to atmospheric pressure P_0 , we find that

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

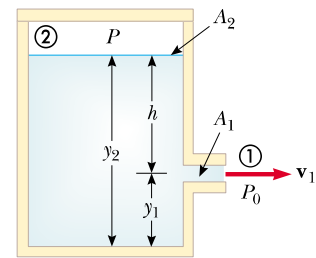


Figure 14.21 (Example 14.10) A liquid leaves a hole in a tank at speed v_1 .

But $y_2 - y_1 = h$; thus, this expression reduces to

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

When P is much greater than P_0 (so that the term $2gh$ can be neglected), the exit speed of the water is mainly a function of P . If the tank is open to the atmosphere, then $P = P_0$ and $v_1 = \sqrt{2gh}$. In other words, for an open tank, the speed of liquid coming out through a hole a distance h below the surface is equal to that acquired by an object falling freely through a vertical distance h . This phenomenon is known as **Torricelli's law**.

What If? What if the position of the hole in Figure 14.21 could be adjusted vertically? If the tank is open to the atmosphere and sitting on a table, what position of the hole would cause the water to land on the table at the farthest distance from the tank?

Answer We model a parcel of water exiting the hole as a projectile. We find the time at which the parcel strikes the table from a hole at an arbitrary position:

$$\begin{aligned} y_f &= y_i + v_{yi}t - \frac{1}{2}gt^2 \\ 0 &= y_1 + 0 - \frac{1}{2}gt^2 \\ t &= \sqrt{\frac{2y_1}{g}} \end{aligned}$$

Thus, the horizontal position of the parcel at the time it strikes the table is

$$\begin{aligned} x_f &= x_i + v_{xi}t = 0 + \sqrt{2g(y_2 - y_1)} \sqrt{\frac{2y_1}{g}} \\ &= 2\sqrt{(y_2y_1 - y_1^2)} \end{aligned}$$

Now we maximize the horizontal position by taking the derivative of x_f with respect to y_1 (because y_1 , the height of the hole, is the variable that can be adjusted) and setting it equal to zero:

$$\frac{dx_f}{dy_1} = \frac{1}{2}(2)(y_2y_1 - y_1^2)^{-1/2}(y_2 - 2y_1) = 0$$

This is satisfied if

$$y_1 = \frac{1}{2}y_2$$

Thus, the hole should be halfway between the bottom of the tank and the upper surface of the water to maximize the horizontal distance. Below this location, the water is projected at a higher speed, but falls for a short time interval, reducing the horizontal range. Above this point, the water is in the air for a longer time interval, but is projected with a smaller horizontal speed.