

Final worked out, January 2015

Q1. A Traffic light at rest

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure 1. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N.

Will the traffic light remain hanging in this situation, or will one of the cables break?

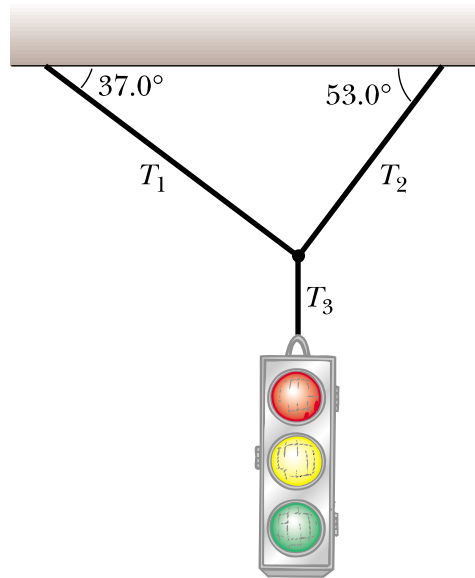


Figure 1.

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure 5.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Will the traffic light remain hanging in this situation, or will one of the cables break?

Solution We *conceptualize* the problem by inspecting the drawing in Figure 5.10a. Let us assume that the cables do not break so that there is no acceleration of any sort in this problem in any direction. This allows us to *categorize* the problem as one of equilibrium. Because the acceleration of the system is zero, we know that the net force on the light and the net force on the knot are both zero. To *analyze* the

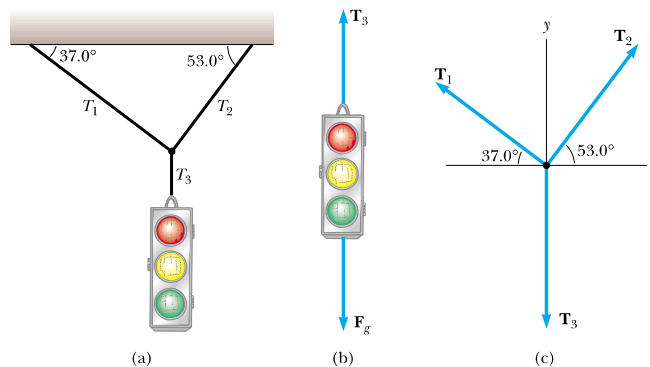


Figure 5.10 (Example 5.4) (a) A traffic light suspended by cables. (b) Free-body diagram for the traffic light. (c) Free-body diagram for the knot where the three cables are joined.

problem, we construct two free-body diagrams—one for the traffic light, shown in Figure 5.10b, and one for the knot that holds the three cables together, as in Figure 5.10c. This knot is a convenient object to choose because all the forces of interest act along lines passing through the knot.

With reference to Figure 5.10b, we apply the equilibrium condition in the y direction, $\Sigma F_y = 0 \rightarrow T_3 - F_g = 0$. This leads to $T_3 = F_g = 122 \text{ N}$. Thus, the upward force \mathbf{T}_3 exerted by the vertical cable on the light balances the gravitational force.

Next, we choose the coordinate axes shown in Figure 5.10c and resolve the forces acting on the knot into their components:

Force	x Component	y Component
\mathbf{T}_1	$-T_1 \cos 37.0^\circ$	$T_1 \sin 37.0^\circ$
\mathbf{T}_2	$T_2 \cos 53.0^\circ$	$T_2 \sin 53.0^\circ$
\mathbf{T}_3	0	-122 N

Knowing that the knot is in equilibrium ($\mathbf{a} = 0$) allows us to write

$$(1) \quad \Sigma F_x = -T_1 \cos 37.0^\circ + T_2 \cos 53.0^\circ = 0$$

$$(2) \quad \Sigma F_y = T_1 \sin 37.0^\circ + T_2 \sin 53.0^\circ + (-122 \text{ N}) = 0$$

From (1) we see that the horizontal components of \mathbf{T}_1 and \mathbf{T}_2 must be equal in magnitude, and from (2) we see that the sum of the vertical components of \mathbf{T}_1 and \mathbf{T}_2 must balance the downward force \mathbf{T}_3 , which is equal in magnitude to

the weight of the light. We solve (1) for T_2 in terms of T_1 to obtain

$$(3) \quad T_2 = T_1 \left(\frac{\cos 37.0^\circ}{\cos 53.0^\circ} \right) = 1.33 T_1$$

This value for T_2 is substituted into (2) to yield

$$T_1 \sin 37.0^\circ + (1.33 T_1)(\sin 53.0^\circ) - 122 \text{ N} = 0$$

$$T_1 = 73.4 \text{ N}$$

$$T_2 = 1.33 T_1 = 97.4 \text{ N}$$

Both of these values are less than 100 N (just barely for T_2), so the cables will not break. Let us *finalize* this problem by imagining a change in the system, as in the following **What If?**

What If? Suppose the two angles in Figure 5.10a are equal. What would be the relationship between T_1 and T_2 ?

Answer We can argue from the symmetry of the problem that the two tensions T_1 and T_2 would be equal to each other. Mathematically, if the equal angles are called θ , Equation (3) becomes

$$T_2 = T_1 \left(\frac{\cos \theta}{\cos \theta} \right) = T_1$$

which also tells us that the tensions are equal. Without knowing the specific value of θ , we cannot find the values of T_1 and T_2 . However, the tensions will be equal to each other, regardless of the value of θ .

Q2. Connected Blocks in Motion

Solve this question symbolically.

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 2. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k . The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

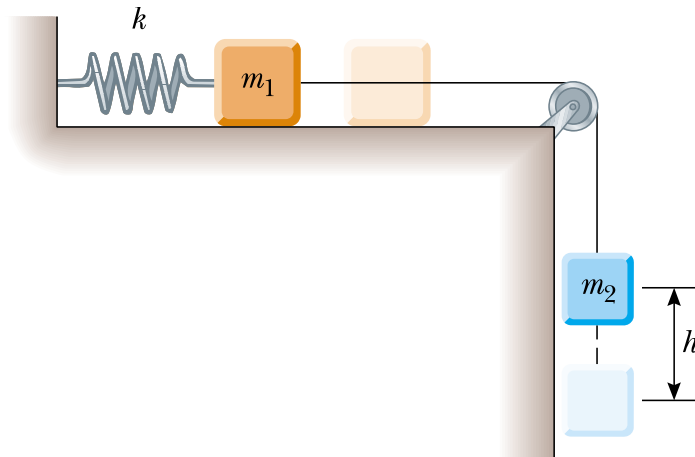


Figure 2.

Example 8.10 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.15. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k . The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

Solution The key word *rest* appears twice in the problem statement. This suggests that the configurations associated with rest are good candidates for the initial and final configurations because the kinetic energy of the system is zero for these configurations. (Also note that because we are concerned only with the beginning and ending points of the motion, we do not need to label events with circled letters as we did in the previous two examples. Simply using *i* and *f* is sufficient to keep track of the situation.) In this situation, the system consists of the two blocks, the spring, and the Earth. We need to consider two forms of potential energy: gravitational and elastic. Because the initial and final kinetic energies of the system are zero, $\Delta K = 0$, and we can write

$$(1) \quad \Delta E_{\text{mech}} = \Delta U_g + \Delta U_s$$

where $\Delta U_g = U_{gf} - U_{gi}$ is the change in the system's gravitational potential energy and $\Delta U_s = U_{sf} - U_{si}$ is the change in the system's elastic potential energy. As the hanging block falls a distance h , the horizontally moving block moves the same distance h to the right. Therefore, using Equation 8.14, we find that the loss in mechanical energy in the system due to friction between the horizontally sliding block and the surface is

$$(2) \quad \Delta E_{\text{mech}} = -f_k h = -\mu_k m_1 g h$$

The change in the gravitational potential energy of the system is associated with only the falling block because the vertical coordinate of the horizontally sliding block does not change. Therefore, we obtain

$$(3) \quad \Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

where the coordinates have been measured from the lowest position of the falling block.

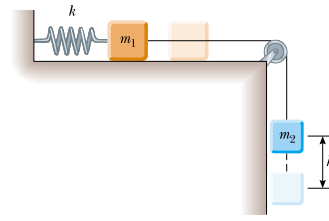


Figure 8.15 (Example 8.10) As the hanging block moves from its highest elevation to its lowest, the system loses gravitational potential energy but gains elastic potential energy in the spring. Some mechanical energy is lost because of friction between the sliding block and the surface.

The change in the elastic potential energy of the system is that stored in the spring:

$$(4) \quad \Delta U_s = U_{sf} - U_{si} = \frac{1}{2} k h^2 - 0$$

Substituting Equations (2), (3), and (4) into Equation (1) gives

$$\begin{aligned} -\mu_k m_1 g h &= -m_2 g h + \frac{1}{2} k h^2 \\ \mu_k &= \frac{m_2 g - \frac{1}{2} k h}{m_1 g} \end{aligned}$$

This setup represents a way of measuring the coefficient of kinetic friction between an object and some surface. As you can see from the problem, sometimes it is easier to work with the changes in the various types of energy rather than the actual values. For example, if we wanted to calculate the numerical value of the gravitational potential energy associated with the horizontally sliding block, we would need to specify the height of the horizontal surface relative to the lowest position of the falling block. Fortunately, this is not necessary because the gravitational potential energy associated with the first block does not change.

Q3 A Two-dimensional elastic collision

Figure 3 shows an elastic collision of two pucks (masses $m_A = 0.500$ kg and $m_B = 0.300$ kg) on a frictionless air-hockey table. Puck A has an initial velocity of 4.00 m/s in the positive x - direction and a final velocity of 2.00 m/s in an unknown direction α . Puck B is initially at rest. Find the final speed v_{B2} of puck B and the angles α and β .

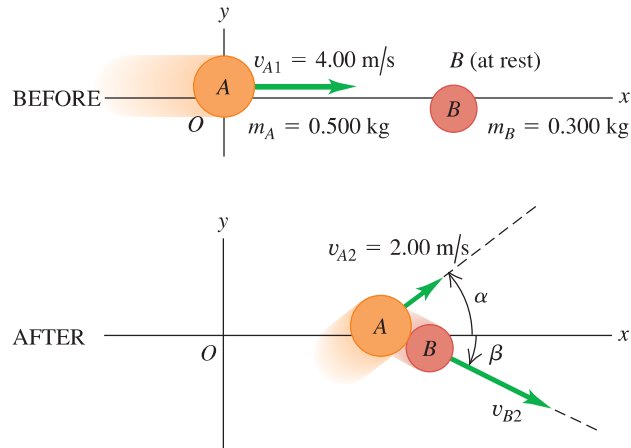


Figure 3.

Example 8.12 A two-dimensional elastic collision

Figure 8.26 shows an elastic collision of two pucks (masses $m_A = 0.500 \text{ kg}$ and $m_B = 0.300 \text{ kg}$) on a frictionless air-hockey table. Puck A has an initial velocity of 4.00 m/s in the positive x -direction and a final velocity of 2.00 m/s in an unknown direction α . Puck B is initially at rest. Find the final speed v_{B2} of puck B and the angles α and β .

SOLUTION

IDENTIFY and SET UP: We'll use the equations for conservation of energy and conservation of x - and y -momentum. These three equations should be enough to solve for the three target variables given in the problem statement.

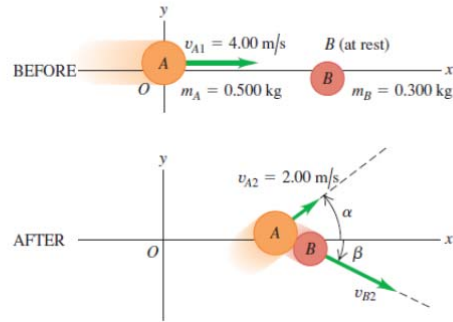
EXECUTE: The collision is elastic, so the initial and final kinetic energies of the system are equal:

$$\begin{aligned}\frac{1}{2}m_A v_{A1}^2 &= \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 \\ v_{B2}^2 &= \frac{m_A v_{A1}^2 - m_A v_{A2}^2}{m_B} \\ &= \frac{(0.500 \text{ kg})(4.00 \text{ m/s})^2 - (0.500 \text{ kg})(2.00 \text{ m/s})^2}{0.300 \text{ kg}} \\ v_{B2} &= 4.47 \text{ m/s}\end{aligned}$$

Conservation of the x - and y -components of total momentum gives

$$\begin{aligned}m_A v_{A1x} &= m_A v_{A2x} + m_B v_{B2x} \\ (0.500 \text{ kg})(4.00 \text{ m/s}) &= (0.500 \text{ kg})(2.00 \text{ m/s})(\cos \alpha) \\ &\quad + (0.300 \text{ kg})(4.47 \text{ m/s})(\cos \beta) \\ 0 &= m_A v_{A2y} + m_B v_{B2y} \\ 0 &= (0.500 \text{ kg})(2.00 \text{ m/s})(\sin \alpha) \\ &\quad - (0.300 \text{ kg})(4.47 \text{ m/s})(\sin \beta)\end{aligned}$$

8.26 An elastic collision that isn't head-on.



These are two simultaneous equations for α and β . We'll leave it to you to supply the details of the solution. (*Hint:* Solve the first equation for $\cos \beta$ and the second for $\sin \beta$; square each equation and add. Since $\sin^2 \beta + \cos^2 \beta = 1$, this eliminates β and leaves an equation that you can solve for $\cos \alpha$ and hence for α . Substitute this value into either of the two equations and solve for β .) The results are

$$\alpha = 36.9^\circ \quad \beta = 26.6^\circ$$

EVALUATE: To check the answers we confirm that the y -momentum, which was zero before the collision, is in fact zero after the collision. The y -momenta are

$$\begin{aligned}p_{A2y} &= (0.500 \text{ kg})(2.00 \text{ m/s})(\sin 36.9^\circ) = +0.600 \text{ kg} \cdot \text{m/s} \\ p_{B2y} &= -(0.300 \text{ kg})(4.47 \text{ m/s})(\sin 26.6^\circ) = -0.600 \text{ kg} \cdot \text{m/s}\end{aligned}$$

and their sum is indeed zero.

Q4 The Venturi Tube

Solve this question symbolically.

The horizontal constricted pipe illustrated in Figure 4, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 if the pressure difference $P_1 - P_2$ is known.

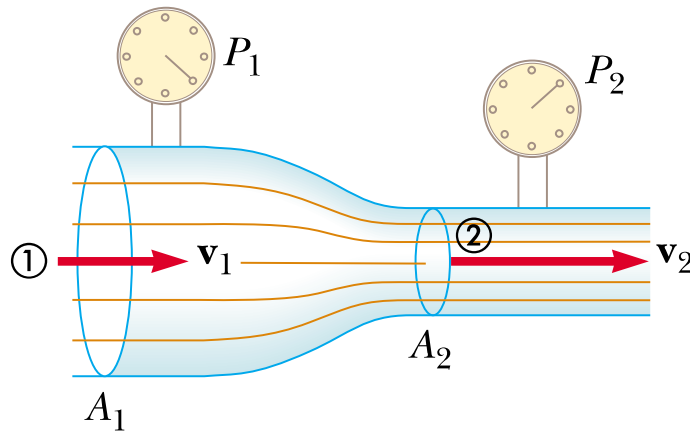


Figure 4.

Example 14.9 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 14.20, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 if the pressure difference $P_1 - P_2$ is known.

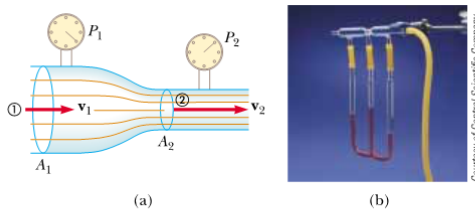


Figure 14.20 (Example 14.9) (a) Pressure P_1 is greater than pressure P_2 because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube, located at the top of the photograph. The higher level of fluid in the middle column shows that the pressure at the top of the column, which is in the constricted region of the Venturi tube, is lower.

Solution Because the pipe is horizontal, $y_1 = y_2$, and applying Equation 14.8 to points 1 and 2 gives

$$(1) \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

From the equation of continuity, $A_1 v_1 = A_2 v_2$, we find that

$$(2) \quad v_1 = \frac{A_2}{A_1} v_2$$

Substituting this expression into Equation (1) gives

$$P_1 + \frac{1}{2}\rho \left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

We can use this result and the continuity equation to obtain an expression for v_1 . Because $A_2 < A_1$, Equation (2) shows us that $v_2 > v_1$. This result, together with Equation (1), indicates that $P_1 > P_2$. In other words, the pressure is reduced in the constricted part of the pipe.

Q5 Two Springs in Parallel or in Series

Given:

- $m = 5 \text{ Kg}$
- $k = 1000\text{N/m}$.

- A) What is the angular frequency ω if the two springs, each with spring constant k , are placed in parallel?
- B) What is the angular frequency ω if the two springs, each with spring constant k , are placed in series?

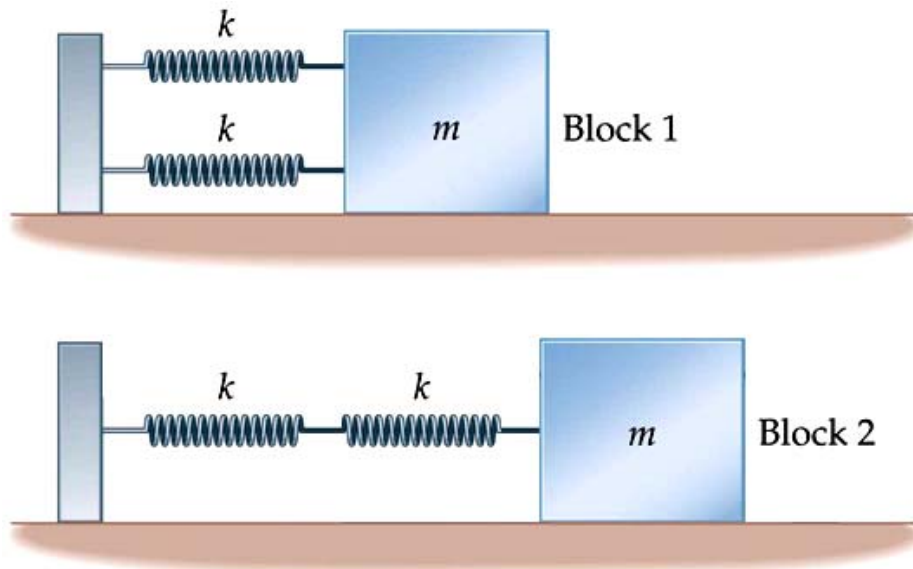


Figure 5.

A) $k_{eq} = 2k$

$$\omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2000}{5}} = 20\text{rad/s}$$

B) $k_{eq} = \frac{1}{2}k$

$$\omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\frac{1}{2} * k}{m}} = \sqrt{\frac{500}{5}} = 10\text{rad/s}$$

Q6. Sound Levels

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is $2.0 \times 10^{-7} \text{ W/m}^2$. Find the sound level heard by the worker.

A) when one machine is operating

B) when both machines are operating.

Example 17.4 Sound Levels

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is $2.0 \times 10^{-7} \text{ W/m}^2$. Find the sound level heard by the worker

(A) when one machine is operating

(B) when both machines are operating.

Solution

(A) The sound level at the location of the worker with one machine operating is calculated from Equation 17.8:

$$\begin{aligned}\beta_1 &= 10 \log \left(\frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log(2.0 \times 10^5) \\ &= 53 \text{ dB}\end{aligned}$$

(B) When both machines are operating, the intensity is doubled to $4.0 \times 10^{-7} \text{ W/m}^2$; therefore, the sound level now is

$$\begin{aligned}\beta_2 &= 10 \log \left(\frac{4.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log(4.0 \times 10^5) \\ &= 56 \text{ dB}\end{aligned}$$

From these results, we see that when the intensity is doubled, the sound level increases by only 3 dB.

What If? *Loudness* is a psychological response to a sound and depends on both the intensity and the frequency of the sound. As a rule of thumb, a doubling in loudness is approximately associated with an increase in sound level of 10 dB. (Note that this rule of thumb is relatively inaccurate at very low or very high frequencies.) If the loudness of the

Q7. The Mistuned Piano Strings

Two identical piano strings of length 0.750 m are each tuned exactly to 440 Hz. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamentals of the two strings?

Example 18.8 The Mistuned Piano Strings

Two identical piano strings of length 0.750 m are each tuned exactly to 440 Hz. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamentals of the two strings?

Solution We find the ratio of frequencies if the tension in one string is 1.0% larger than the other:

$$\begin{aligned}\frac{f_2}{f_1} &= \frac{(v_2/2L)}{(v_1/2L)} = \frac{v_2}{v_1} = \frac{\sqrt{T_2/\mu}}{\sqrt{T_1/\mu}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1.010T_1}{T_1}} \\ &= 1.005\end{aligned}$$

Thus, the frequency of the tightened string is

$$f_2 = 1.005f_1 = 1.005(440 \text{ Hz}) = 442 \text{ Hz}$$

and the beat frequency is

$$f_{\text{beat}} = 442 \text{ Hz} - 440 \text{ Hz} = \mathbf{2 \text{ Hz.}}$$

Q8. A Solar Cooker

Given for water:

- The specific heat is $4186 \text{ J/Kg}\cdot\text{K}$.
- The Heat of Fusion is $3.34 \times 10^5 \text{ J/Kg}$.
- The Heat of Vaporization is $2.26 \times 10^6 \text{ J/Kg}$.
- The specific density is 1000 Kg/m^3 .

A solar cooker consists of a curved reflecting surface that concentrates sunlight onto the object to be warmed (Figure 6). The solar power per unit area reaching the Earth's surface at the location is 600 W/m^2 . The cooker faces the Sun and has a diameter of 0.600 m . Assume that 40.0% of the incident energy is transferred to 0.500 L of water in an open container, initially at 20.0°C . How long does it take to completely boil away the water? (Ignore the heat capacity of the container.)



Figure 6.

The power incident on the solar collector is

$$P_i = IA = [600 \text{ W/m}^2][\pi(0.300 \text{ m})^2] = 170 \text{ W}.$$

For a 40.0% reflector, the collected power is $P_c = 67.9 \text{ W}$. The total energy required to increase the temperature of the water to the boiling point and to evaporate it is $Q = cm\Delta T + mL_v$:

$$Q = 0.500 \text{ kg}[4186 \text{ J/kg}\cdot^\circ\text{C}(80.0^\circ\text{C}) + 2.26 \times 10^6 \text{ J/kg}] = 1.30 \times 10^6 \text{ J}.$$

$$\text{The time interval required is } \Delta t = \frac{Q}{P_c} = \frac{1.30 \times 10^6 \text{ J}}{67.9 \text{ W}} = \boxed{5.31 \text{ h}}.$$

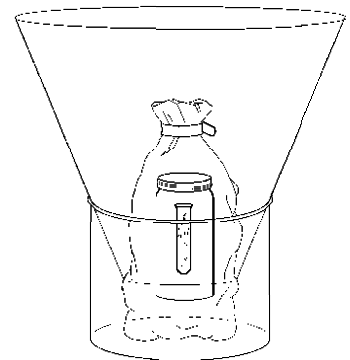


FIG. P20.63

Q9 A Multiloop Circuit

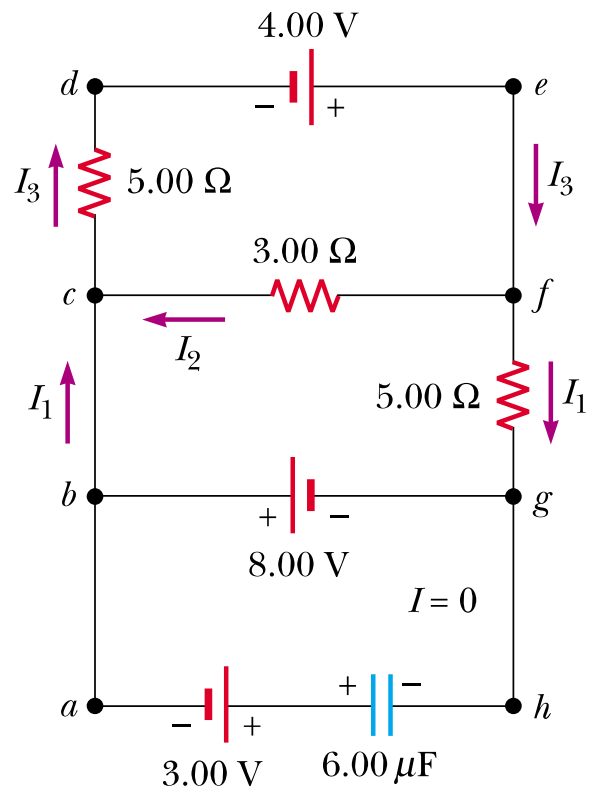


Figure 7.

Under steady-state conditions, find the unknown currents I_1 , I_2 , and I_3 in the multiloop circuit shown in Figure 7.

- A) $I_1 =$
- B) What is the charge on the capacitor?

Example 28.10 A Multiloop Circuit

(A) Under steady-state conditions, find the unknown currents I_1 , I_2 , and I_3 in the multiloop circuit shown in Figure 28.18.

Solution First note that because the capacitor represents an open circuit, there is no current between g and b along path $ghab$ under steady-state conditions. Therefore, when the charges associated with I_1 reach point g , they all go toward point b through the 8.00-V battery; hence, $I_{gb} = I_1$. Labeling the currents as shown in Figure 28.18 and applying Equation 28.9 to junction c , we obtain

$$(1) \quad I_1 + I_2 = I_3$$

Equation 28.10 applied to loops $defcd$ and $cfghc$, traversed clockwise, gives

$$(2) \quad \text{defcd} \quad 4.00 \text{ V} - (3.00 \, \Omega)I_2 - (5.00 \, \Omega)I_3 = 0$$

$$(3) \quad \text{cfghc} \quad (3.00 \, \Omega)I_2 - (5.00 \, \Omega)I_1 + 8.00 \text{ V} = 0$$

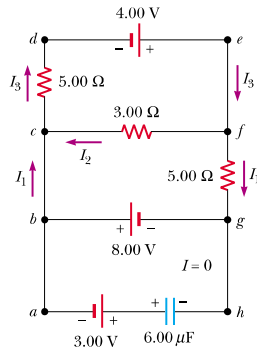


Figure 28.18 (Example 28.10) A multiloop circuit. Kirchhoff's loop rule can be applied to *any* closed loop, including the one containing the capacitor.

From Equation (1) we see that $I_1 = I_3 - I_2$, which, when substituted into Equation (3), gives

$$(4) \quad (8.00 \, \Omega)I_2 - (5.00 \, \Omega)I_3 + 8.00 \text{ V} = 0$$

Subtracting Equation (4) from Equation (2), we eliminate I_3 and find that

$$I_2 = -\frac{4.00 \text{ V}}{11.0 \, \Omega} = -0.364 \text{ A}$$

Because our value for I_2 is negative, we conclude that the direction of I_2 is from e to f in the 3.00- Ω resistor. Despite this interpretation of the direction, however, we must continue to use this negative value for I_2 in subsequent calculations because our equations were established with our original choice of direction.

Using $I_2 = -0.364 \text{ A}$ in Equations (3) and (1) gives

$$I_1 = 1.38 \text{ A} \quad I_3 = 1.02 \text{ A}$$

(B) What is the charge on the capacitor?

Solution We can apply Kirchhoff's loop rule to loop $bghab$ (or any other loop that contains the capacitor) to find the potential difference ΔV_{cap} across the capacitor. We use this potential difference in the loop equation without reference to a sign convention because the charge on the capacitor depends only on the magnitude of the potential difference. Moving clockwise around this loop, we obtain

$$-8.00 \text{ V} + \Delta V_{\text{cap}} - 3.00 \text{ V} = 0$$

$$\Delta V_{\text{cap}} = 11.0 \text{ V}$$

Because $Q = C \Delta V_{\text{cap}}$ (see Eq. 26.1), the charge on the capacitor is

$$Q = (6.00 \, \mu\text{F})(11.0 \text{ V}) = 66.0 \, \mu\text{C}$$

Why is the left side of the capacitor positively charged?

Q10 Reflection and refraction

In Figure 8, material *a* is water and material *b* is glass with index of refraction 1.52. The incident ray makes an angle of 60.0° with the normal;

- A) Find the direction θ_r of the reflected ray
- B) Find the direction θ_b of the refracted ray.

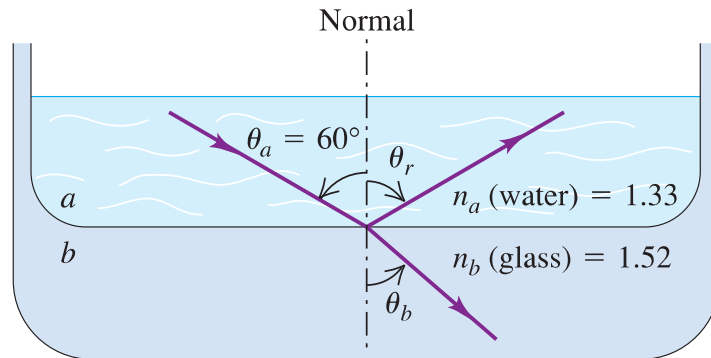


Figure 8

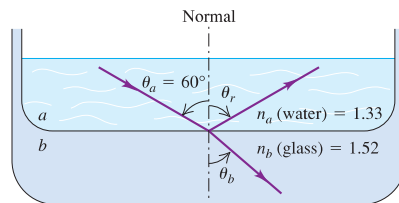
Example 33.1 Reflection and refraction

In Fig. 33.11, material *a* is water and material *b* is glass with index of refraction 1.52. The incident ray makes an angle of 60.0° with the normal; find the directions of the reflected and refracted rays.

SOLUTION

IDENTIFY and SET UP: This is a problem in geometric optics. We are given the angle of incidence $\theta_a = 60.0^\circ$ and the indexes of

33.11 Reflection and refraction of light passing from water to glass.



refraction $n_a = 1.33$ and $n_b = 1.52$. We must find the angles of reflection and refraction θ_r and θ_b ; to do this we use Eqs. (33.2) and (33.4), respectively. Figure 33.11 shows the rays and angles; n_b is slightly greater than n_a , so by Snell's law [Eq. (33.4)] θ_b is slightly smaller than θ_a , as the figure shows.

EXECUTE: According to Eq. (33.2), the angle the reflected ray makes with the normal is the same as that of the incident ray, so $\theta_r = \theta_a = 60.0^\circ$.

To find the direction of the refracted ray we use Snell's law, Eq. (33.4):

$$\begin{aligned} n_a \sin \theta_a &= n_b \sin \theta_b \\ \sin \theta_b &= \frac{n_a}{n_b} \sin \theta_a = \frac{1.33}{1.52} \sin 60.0^\circ = 0.758 \\ \theta_b &= \arcsin(0.758) = 49.3^\circ \end{aligned}$$

EVALUATE: The second material has a larger refractive index than the first, as in Fig. 33.8a. Hence the refracted ray is bent toward the normal and $\theta_b < \theta_a$.