Assignment 2019-Feb Q1 (20 points)

On the truss structure, as shown in the figure below, a horizontal force F acts at point D. All trusses have the same Young's Modulus, equal to E. The cross-sectional area of the diagonal trusses equal $A\sqrt{2}$, and the horizontal and vertical trusses have a cross-sectional area equal to A. Normally, you should determine the truss forces by the method of sections, the method of joints, or a Cremona-Maxwell diagram. In this case, this has been carried out for you, and the truss forces equal:

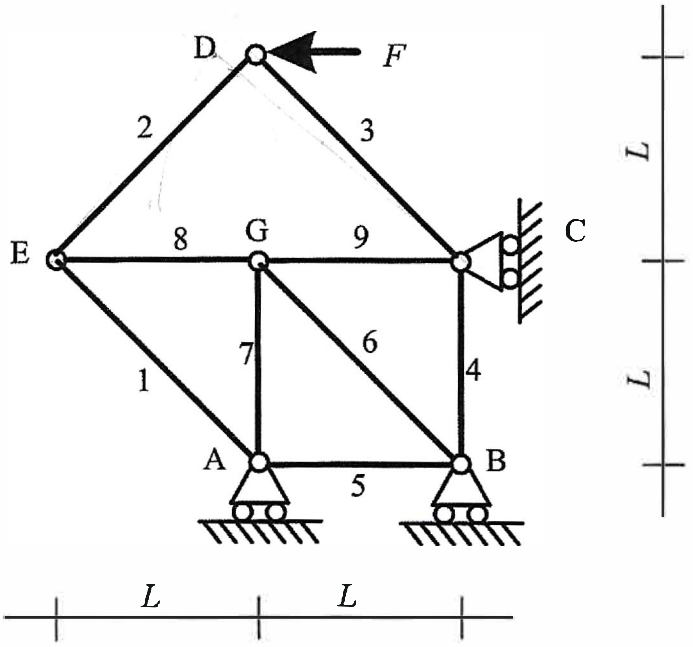
$$F_1 = F_2 = -\frac{1}{2}F\sqrt{2}$$

$$F_3 = F_6 = +\frac{1}{2}F\sqrt{2}$$

$$F_4 = F_9 = +\frac{1}{2}F$$

$$F_5 = F_7 = -\frac{1}{2}F$$

$$F_8 = +F$$



Determine the vertical and horizontal displacement of point D via a Williot diagram. For this you can use the first two pages of scrap paper. Start with point A as indicated on the paper.

Use the following scale for the diagram: $\frac{1}{2}\frac{FL}{EA} \equiv 10\,\text{mm} \equiv 2$ squares

MC1 (3 points)

Given the information above, what is the change of length of truss 3? This is a multiple choice question, which has a single correct answer.

A) $\Delta l_{DC} = +\frac{FL}{EA}$ B) $\Delta l_{DC} = +\frac{1}{2}\frac{FL}{EA}\sqrt{2}$ C) $\Delta l_{DC} = -\frac{FL}{EA}$

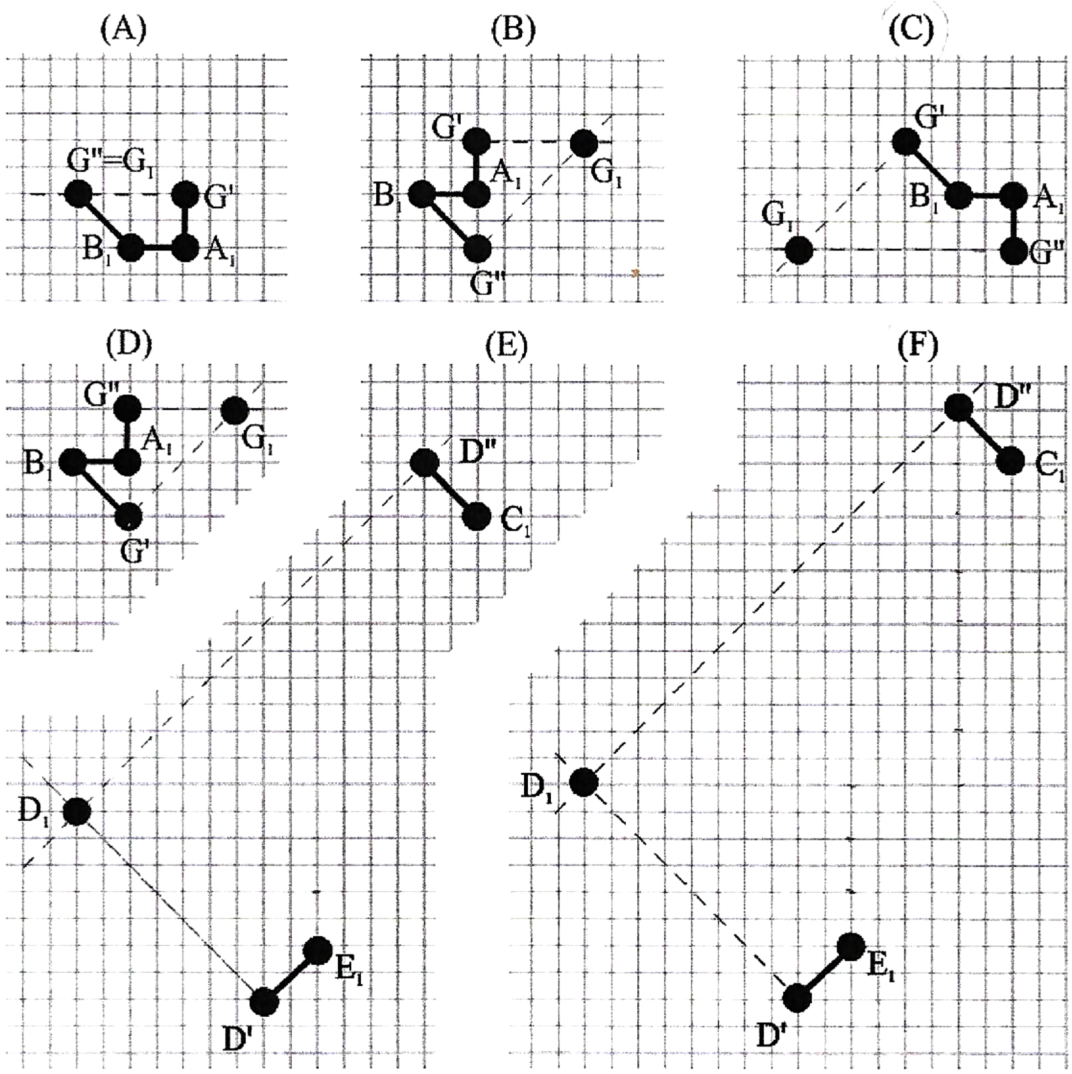
D) $\Delta l_{DC} = +\frac{1}{2}\frac{FL}{EA}$ E) $\Delta l_{DC} = -\frac{1}{2}\frac{FL}{EA}$ F) $\Delta l_{GC} = +\frac{1}{2}\frac{FL\sqrt{2}}{EA\sqrt{2}}$

MC2 (3 points)
Given the information above, what is the change of length of truss 5? This is a multiple choice question, which has a single correct answer.

A) $\Delta l_{AB} = -\frac{1}{2}\frac{FL}{EA}$ B) $\Delta l_{AB} = 0$ (zero stress bar due to 2 rollers) C) $\Delta l_{AB} = +\frac{1}{2}\frac{FL}{EA}$

D) Indeterminate (due to 2 rollers) E) $\Delta l_{AB} = \frac{FL}{EA}$ F) $\Delta l_{DC} = -\frac{1}{2}\frac{FL}{EA}\sqrt{2}$

MR3 (10 points)
Which of the parts below are part of the correct Willot diagram? For all parts below, dotted lines start in a point only in the direction in which they are needed, and of course it is arbitrarily whether a point is G' or G", taking G as an example. Note that this is a multiple response question, which has a single or multiple correct options, and has at least 1 incorrect option (so here 1, 2, 3, 4, or 5 options may be correct).

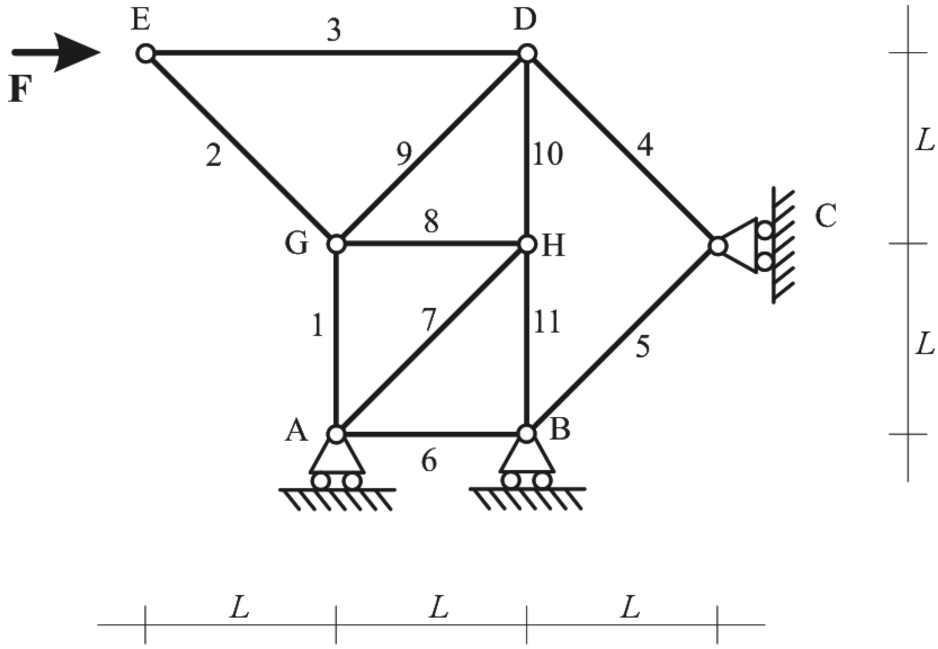


Assignment 2018-Jan Q2 (15 points)

The structure below is supported at A, B, and C, and loaded by a load F at point E. All trusses have the same Young's modulus equal to E. The internal forces in the trusses are given as follows:

$$F_1 = +\frac{1}{2}F, \quad F_2 = F_{10} = 0, \quad F_3 = -F, \quad F_4 = F_5 = -\frac{1}{2}\sqrt{2}F, \quad F_6 = F_8 = F_{11} = -\frac{1}{2}F,$$

$$F_7 = F_9 = +\frac{1}{2}\sqrt{2}F$$



Using the start made for you on the scrap-paper, complete the Williot diagram for this structure. Note that the area of each truss has been designed such that the value of the normal stress in each truss equals σ, with an exception for the unloaded trusses, which keep their zero stresses. Furthermore, the scale used in the Williot diagram equals $\sigma L/E$=2 squares.

MC9 (2 points)
Given the information above, what is the value of the shortening of truss 3:

A) $\Delta l_{ED} = \dfrac{\sigma 2L}{AE}$ B) $\Delta l_{ED} = 3\dfrac{\sigma 2L}{AE}$ C) $\Delta l_{ED} = \dfrac{2L}{EA}$

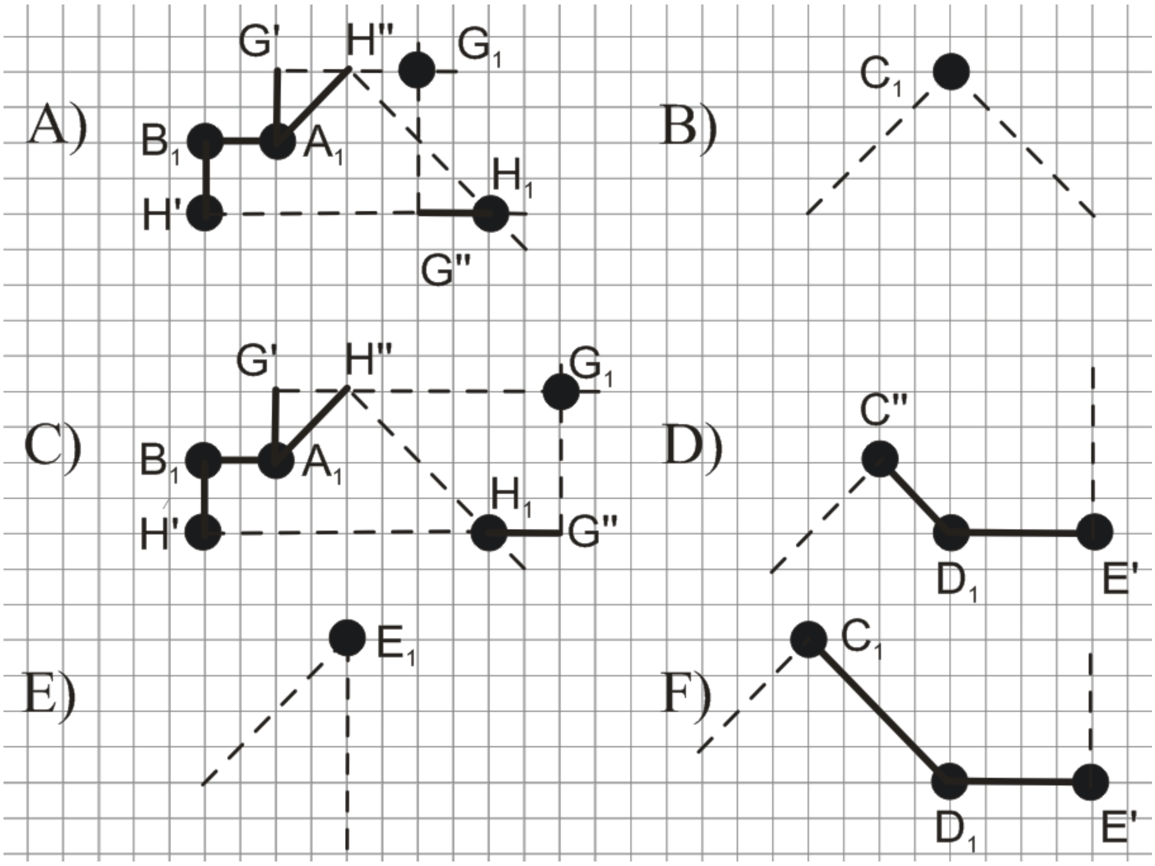
D) $\Delta l_{ED} = \dfrac{1}{2}\sqrt{2}\dfrac{\sigma L}{E}$ E) $\Delta l_{ED} = 2\dfrac{\sigma L}{E}$ F) $\Delta l_{ED} = \dfrac{FL}{EA}$

MC10 (2 points)
Given the information above, what is the value of the shortening of truss 9:

A) $\Delta l_{GD} = +1/2\sqrt{2}\dfrac{\sigma\sqrt{2}L}{E}$ B) $\Delta l_{GD} = \dfrac{\sigma\sqrt{2}L}{E}$ C) $\Delta l_{GD} = +1/2\sqrt{2}\dfrac{\sigma\sqrt{(2L)}}{E}$

D) $\Delta l_{GD} = \dfrac{\sigma\sqrt{2}L}{A\sqrt{2}E}$ E) $\Delta l_{GD} = \dfrac{(F/A)L}{\sqrt{2}AE}$ F) $\Delta l_{GD} = 2\sqrt{2}\dfrac{\sigma L}{E}$

MR11 (7 points)
Which of the parts below are part of the correct Willot diagram? Dotted lines start in a point only in the direction in which they are needed and of course it is arbitrarily whether a point is G' or G'', taking G as an example. Note that this is a multiple response answer, which may have zero, a single, or multiple correct answers.



MC12 (2 points)
What is the horizontal displacement of point E:

A) $\overrightarrow{w_{EH}} = 4\frac{\sigma L}{E}$ B) $\overrightarrow{w_{EH}} = 7\frac{\sigma L}{E}$ C) $\overrightarrow{w_{EH}} = 10\frac{\sigma L}{E}$

D) $\overleftarrow{w_{EH}} = \frac{\sigma L}{E}$ E) $\overrightarrow{w_{EH}} = 2\frac{\sigma L}{E}$ F) $\overrightarrow{w_{EH}} = 8\frac{\sigma L}{E}$

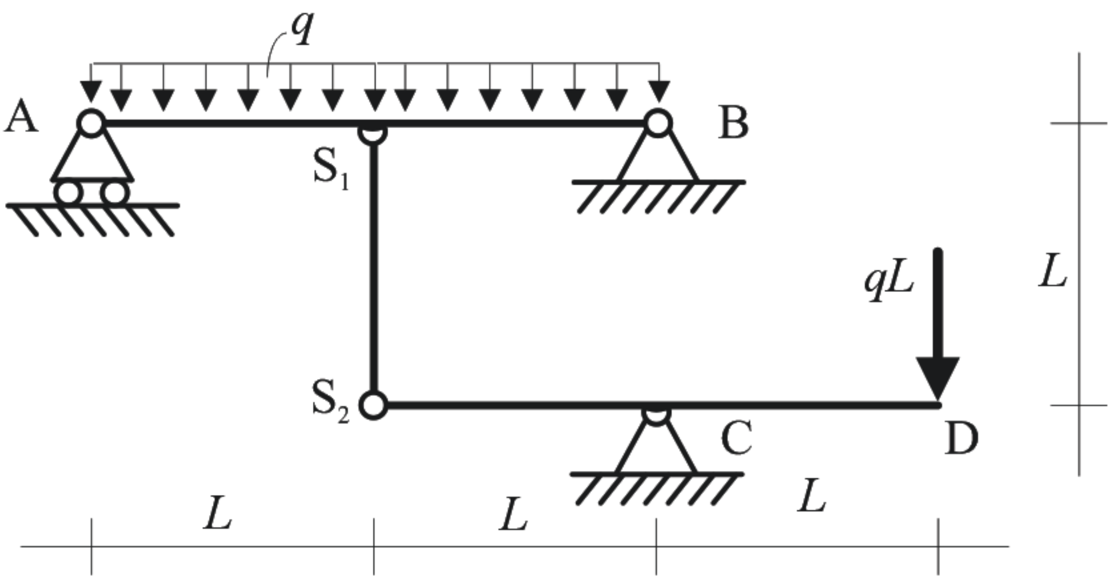
MC13 (2 points)
What is the vertical displacement of point E:

A) $w_{EV} \downarrow = 3\frac{\sigma L}{E}$ B) $w_{EV} \uparrow = 7\frac{\sigma L}{E}$ C) $w_{EV} \uparrow = 3\frac{\sigma L}{E}$

D) $w_{EV} \downarrow = 7\frac{\sigma L}{E}$ E) $w_{EV} \uparrow = 7\sqrt{2}\frac{\sigma L}{E}$ F) $w_{EV} \uparrow = 3\frac{1}{2}\sqrt{2}\frac{\sigma L}{E}$

Assignment 2018-Jan Q3 (15 points)

Given is the structure in the figure below. The bending stiffness of all elements equals EI and normal and shear deformations can be neglected for all elements.



MR14 (5 points)
If it is asked to calculate the displacements of point S1 and D, then which of the strategies (or part of strategies) below are correct to find these displacements. Note that this is a multiple response answer, which may have zero, a single, or multiple correct answers.

A) First the downwards displacement of S1 only due to the distributed load can be calculated via an additional standard beam equation. This displacement is transferred to S2 (as there are no normal deformations). The downwards displacement of D is then equal to the displacement of S2 plus a component due to bending in beam S2-D, to be calculated by an additional standard beam equation.
B) Using superposition, first the displacements of S1 and D can be calculated due to only the distributed load, then the displacements of S1 and D due to only the concentrated load, and finally the displacements due to the two loads should be summed up.
C) Calculating the normal force in S1-S2, beam A-B can be taken apart, the normal force put as an external force on its middle, and virtually clamped at A under an unknown angle and released at B. Then using standard beam equations, the displacement of B can be calculated, to be put only then to zero. As such the angle of A is found, followed by the displacement of S1.
D) Excluding parts A-B and S1-S2, the normal force in part S1-S2 can be calculated by equilibrium of part S2-D.
E) The so-call "inclined beam effect" needs not to be used in this exercise, as the additional standard beam equations, which should be used anyway, take care of these effects.
F) At some stage of the derivations, the load qL need to be seen as a load qL and a bending moment qL^2 at C, otherwise it is not possible to calculate the displacements of D.

MC15 (1 points)
Calculate the normal force in beam S1-S2 and indicate the correct answer among the answers below.

A) $F_{S1-S2} = -qL$ B) $F_{S1-S2} = -\left(qL + \frac{1}{2}qL\right)$ C) $F_{S1-S2} = -\left(qL + qL\right)$

D) $F_{S1-S2} = -\left(qL - \frac{1}{2}qL\right)$ E) $F_{S1-S2} = -3qL$ F) $F_{S1-S2} = -\left(2qL + \frac{1}{2}qL\right)$

MC16 (4 points)
Assume that the normal force in S1-S2 equals -3qL. Note that this needs not to be the correct answer of MC15. Calculate the downwards displacement of S1 and pick the correct answer below.

A) $w_{S1} \downarrow = \frac{80}{384}\frac{qL^4}{EI}$

B) $w_{S1} \downarrow = -\frac{5}{384}\frac{q(2L)^3}{EI} + \frac{(3qL)L^3}{48EI}$

C) $w_{S1} \downarrow = \frac{80}{384}\frac{qL^4}{EI} - \frac{(3qL)L^3}{48EI} + \varphi_A \circlearrowright \frac{L}{2} = \frac{80}{384}\frac{qL^4}{EI} - \frac{(3qL)L^3}{48EI} + \frac{qL^3}{24EI}\frac{L}{2}$

D) $w_{S1} \downarrow = \frac{5}{384}\frac{q(2L)^3}{EI} - \frac{(3qL)L^3}{48EI}$

E) $w_{S1} \downarrow = \frac{5}{384}\frac{q(2L)^4}{EI} - \frac{(3qL)(2L)^3}{48EI}$

F) $w_{S1} \uparrow = \frac{(3qL)L^3}{48EI} + \varphi_A \circlearrowright \frac{L}{2} - \frac{80}{384}\frac{qL^4}{EI} = \frac{(3qL)L^3}{48EI} + \frac{qL^3}{24EI}\frac{L}{2} - \frac{80}{384}\frac{qL^4}{EI}$

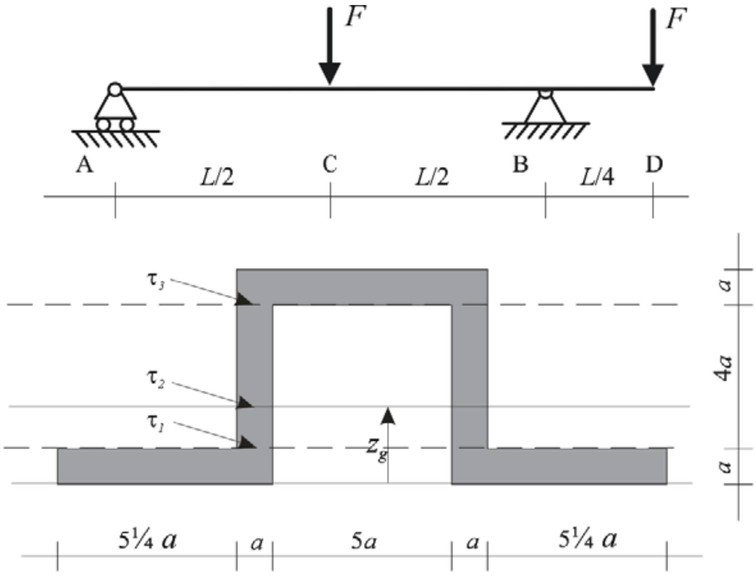
MC17 (5 points)
Now assume that the displacement of S1 (and thus S2) equals zero (0). Note that this value needs not to be related to the previous questions. Calculate the displacement of D and find the correct answer below:

A) $w_{DV} \downarrow = \frac{1}{6}\frac{qL^3}{EI}L + \frac{qLL^3}{3EI}$ B) $w_{DV} \downarrow = \frac{1}{6}\frac{qL^3}{EI}L - \frac{qLL^3}{3EI}$ C) $w_{DV} \downarrow = \frac{2}{3}\frac{qL^4}{EI}$

D) $w_{DV} \downarrow = \frac{1}{6}\frac{qL^3}{EI}L + \frac{qLL^3}{3EI}L$ E) $w_{DV} \downarrow = \frac{1}{3}\frac{qL^3}{EI}L$ F) $w_{DV} \downarrow = \frac{qLL^3}{3EI}$

Assignment 2018-Jan Q1 (20 points)

In the figure below you will find a beam, supported with a roller on the left and a hinge at B, and loaded by two concentrated forces F. The cross-section of the beam is found at the bottom of the figure, with all dimensions given parametrically by a.



Cross-section beam AD

MC1 (1 points)
Calculate the total area of the cross-section, find the correct answer below and mark the correct answer on the Applied Mechanics Multiple Choice (AMMC) sheet.

A) $25\frac{1}{2}a^3$ B) $31\frac{1}{2}a^2$ C) $31\frac{1}{2}a^3$

D) $25\frac{1}{2}a^2$ E) $27\frac{1}{2}a^2$ F) $28.5a^2$

MC2 (3 points)
Derive the first moment of area relative to the bottom of the cross-section (hint: this implies each area multiplied by the distance of its own line of gravity to the bottom line of the cross-section). Mark the correct answer below on the AMMC.

A) $66\frac{1}{8}a^2$ B) $68.75a^3$ C) $66\frac{1}{8}a^3$

D) $41\frac{1}{4}a^2$ E) $63\frac{1}{2}a^3$ F) $32.75a^3$

MC3 (2 points)
Calculate the position of the horizontal line of gravity relative to the bottom line of the cross-section z_g, as also shown in the figure:

A) $\left(66\frac{1}{8}a^3\right)/\left(25\frac{1}{2}a^2\right)$ B) $\dfrac{\text{Correct answer MC1}}{\text{Correct answer MC2}}$ C) $\left(25\frac{1}{2}a^3\right)/\left(66\frac{1}{8}a^2\right)$

D) $3a$ E) $3a-2.5a$ F) $2.5a$

MC4 (2 points)
Now assume z_g equals $3a$. Note that this needs not to be the correct answer for MC3! The contribution of the top flange ($7a \times a$) to the second moment of area of the complete cross-section about the horizontal line of gravity equals:

A) $\frac{1}{12}a*(7a)^3+(a*7a)\left(3a-\frac{1}{2}a\right)^2$ B) $\frac{1}{12}7a*a^3+(7a*a)\left(3a-\frac{1}{2}a\right)^2$

C) $\frac{1}{12}a*(7a)^3-(a*7a)^2\left(3a+\frac{1}{2}a\right)^2$ D) $\frac{1}{12}7a^3*a^3+\left(7a^2*a^2\right)\left(3a+\frac{1}{2}a\right)$

E) $\frac{1}{12}7^3a^3*a^3-\left(7^2a^2*a^2\right)\left(3a-\frac{1}{2}a\right)$ F) $\frac{1}{12}7a*a^3+(7a*a)\left(5\frac{1}{2}a\right)^2$

MC5 (4 points)
Now assume z_g equals $2.5a$. Note that this needs not to be the correct answer for MC3. Derive the second moment of area I about the horizontal line of gravity. Find the correct answer below.

A) $\frac{3055}{24}a^4$ B) $\frac{3034}{24}a^4$ C) $\frac{2047}{24}a^4$

D) $\frac{3045}{24}a^4$ E) $\frac{2983}{24}a^4$ F) 82.29167

MC6 (2 points)
Calculate the reaction forces on the beam. These equal:

A) $R_{AV}\uparrow=\frac{5}{4}F, R_{BV}\uparrow=\frac{3}{4}F$ B) $R_{AV}\downarrow=\frac{1}{4}F, R_{BV}\uparrow=\frac{7}{4}F$

C) $R_{AV}\uparrow=\frac{1}{2}F, R_{BV}\uparrow=\frac{7}{4}F$ D) $R_{AV}\uparrow=\frac{1}{4}F, R_{BV}\uparrow=\frac{7}{4}F$

E) $R_{AV}\downarrow=\frac{1}{8}F, R_{BV}\uparrow=\frac{15}{8}F$ F) $R_{AV}\uparrow=\frac{7}{4}F, R_{BV}\uparrow=\frac{1}{4}F$

MC7 (2 points)
The maximum bending moment equals (1/4)FL and is located at B. Assuming I=(2047/24)a^4 and z_g= 2.5a (Note that these need not to be correct answers for previous questions), the maximal tension (+) bending stress σ_{bt} and the maximal compression (-) bending stress σ_{bc} equal:

A) $\sigma_{bt}=+\frac{15}{2047}\frac{FL}{a^3}, \sigma_{bc}=-\frac{21}{2047}\frac{FL}{a^3}$ B) $\sigma_{bt}=+\frac{84}{2047}\frac{FL}{a^3}, \sigma_{bc}=-\frac{60}{2047}\frac{FL}{a^3}$

C) $\sigma_{bt}=+\frac{21}{2047}\frac{FL}{a^3}, \sigma_{bc}=-\frac{15}{2047}\frac{FL}{a^3}$ D) $\sigma_{bt}=-\frac{21}{2047}\frac{FL}{a^3}, \sigma_{bc}=+\frac{15}{2047}\frac{FL}{a^3}$

E) $\sigma_{bt}=+\frac{36}{2047}\frac{FL}{a^3}, \sigma_{bc}=-\frac{36}{2047}\frac{FL}{a^3}$ F) $\sigma_{bt}=-\frac{36}{2047}\frac{FL}{a^3}, \sigma_{bc}=+\frac{51}{2047}\frac{FL}{a^3}$

MR8 (4 points)

The maximum shear force in the beam equals F. Again assuming I=(2047/24)a^4 and z_g= 2.5a (Note that these need not to be correct answers for previous questions), calculate τ_1, τ_2, and τ_3 at the locations as shown in the figure, at the location along the length of this maximum shear force. Which of the statements below are correct? Note that this is a multiple response answer, which may have zero, a single, or multiple correct answers.

A) τ_3 is smaller than τ_1

B) All shear stresses are equal

C) τ_2 is the maximum shear stress in the beam

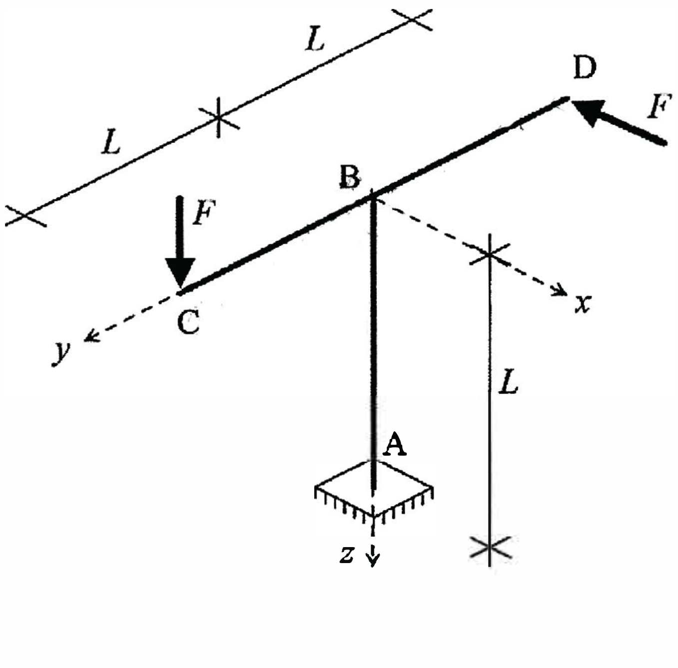
D) τ_1 and τ_3 work in opposite directions

E) τ_1 is smaller than τ_2

F) $\tau_{\max} = \dfrac{372}{2047}\dfrac{F}{a^2}$

Assignment 2019-Feb Q2 (30 points)

The T-shaped structure in the figure below is positioned in the *y-z* plane. The coordinate system *x-y-z* is orthogonal. In point A the structure is completely clamped. In point C, a vertical force F acts in the positive *z*-direction, and in point D a force F works in negative *x*-direction. The bending stiffness about each axis of each part equals EI, and the torsional stiffness equals GI_p (also for each part). **Normal and shear deformations can be neglected.** The final goal of this assignment is to determine the displacements of point C and D.



MR7 (5 points)

Exclude part CBD and calculate the rotations and displacements of B. Indicate the correct answers below. Note that this is a multiple response question, which has a single or multiple correct options, and has at least 1 incorrect option (so here 1, 2, 3, 4, or 5 options may be correct).

A) $\varphi_{Bx}\,{}^{y}_{z}\!\circlearrowright = \frac{FLL}{EI}$

B) $\varphi_{By}\circlearrowleft{}^{z}_{x} = \frac{FLL^2}{2EI}$

C) $\varphi_{By}\circlearrowleft{}^{x}_{z} = \frac{FL^2}{2EI}$

D) $w_{Bx} = \frac{FL^3}{3EI}$

E) $w_{Bx} = -\frac{FL^3}{3EI}$

F) $w_{By} = \frac{FLL}{2EA}$

MR8 (5 points)

Now fictitiously and completely clamp the structure at B, and find the displacements at C. Indicate the correct answers below. Note that this is a multiple response question, which has a single or multiple correct options, and has at least 1 incorrect option (so here 1, 2, 3, 4, or 5 options may be correct).

A) $w_{Cx} = w_{Bx} + \varphi_{Bz}\,{}^{x}_{y}\!\circlearrowright L$

B) $w_{Cy} = w_{By} + 0$

C) $w_{Cz} = w_{Bz} + \varphi_{Bx}\,{}^{y}_{z}\!\circlearrowright L + \frac{FL^3}{3EI}$

D) $w_{Cx} = w_{Bx} + \varphi_{Bz}\,{}^{x}_{y}\!\circlearrowright L + \frac{F(2L)^3}{EI}$

E) $w_{Cy} = w_{By} + \varphi_{By}\,{}^{x}_{z}\!\circlearrowright L + 0$

F) $w_{Cz} = \varphi_{Bx}\,{}^{y}_{z}\!\circlearrowright L + \frac{FL^3}{3EI}$

MR9 (5 points)

Now fictitiously and completely clamp the structure at B, and find the displacements at D. Indicate the correct answers below. Note that this is a multiple response question, which has a single or multiple correct options, and has at least 1 incorrect option (so here 1, 2, 3, 4, or 5 options may be correct).

A) $w_{Dx} = w_{Bx} - \varphi_{Bz} \circlearrowleft^{y}_{x} L - \dfrac{FL^3}{3EI}$ B) $w_{Dy} = w_{By} + 0$ C) $w_{Dz} = w_{Bz} - \varphi_{Bx} \circlearrowright^{y}_{z} L$

D) $w_{Dx} = w_{Bx} - \varphi_{Bz} \circlearrowleft^{x}_{y} L + \dfrac{FL^3}{3EI}$ E) $w_{Dx} = w_{Bx} - \varphi_{Bz} \circlearrowleft^{x}_{y} L - \dfrac{F(2L)^3}{3EI}$ F) $w_{Dy} = w_{By} + \varphi_{By} \circlearrowright^{x}_{z} L + 0$

MC10 (2 points)
What are the final solutions for the displacement of point C in *x,y,* and *z*-direction respectively? This is a multiple choice question, which has a single correct answer.

A) $\dfrac{1}{3}\dfrac{FL^3}{EI} - \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, -\dfrac{4}{3}\dfrac{FL^3}{EI}$ B) $\dfrac{1}{2}\dfrac{FL^3}{EI} - \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, -\dfrac{4}{3}\dfrac{FL^3}{EI}$

C) $\dfrac{1}{2}\dfrac{FL^3}{EI} - \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, -\dfrac{5}{2}\dfrac{FL^3}{EI}$ D) $-\dfrac{2}{3}\dfrac{FL^3}{EI} - \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, -\dfrac{FL^3}{EI}$

E) $-\dfrac{1}{3}\dfrac{FL^3}{EI}, \dfrac{1}{2}\dfrac{FL^3}{EI}, \dfrac{4}{3}\dfrac{FL^3}{EI} + \dfrac{FL^3}{GI_p}$ F) $-\dfrac{1}{3}\dfrac{FL^3}{EI} + \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, \dfrac{4}{3}\dfrac{FL^3}{EI}$

MC11 (2 points)
What are the final solutions for the displacement of point D in *x,y,* and *z*-direction respectively? This is a multiple choice question, which has a single correct answer.

A) $-\dfrac{2}{3}\dfrac{FL^3}{EI} - \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, -\dfrac{1}{2}\dfrac{FL^3}{EI}$ B) $\dfrac{2}{3}\dfrac{FL^3}{EI} - \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, -\dfrac{FL^3}{EI}$

C) $-\dfrac{1}{6}\dfrac{FL^3}{EI} + \dfrac{FL^3}{GI_p}, -\dfrac{1}{6}\dfrac{FL^3}{EI}, \dfrac{4}{3}\dfrac{FL^3}{EI}$ D) $-\dfrac{2}{3}\dfrac{FL^3}{EI} - \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, -\dfrac{FL^3}{EI}$

E) $-\dfrac{1}{3}\dfrac{FL^3}{EI} + \dfrac{FL^3}{GI_p}, -\dfrac{1}{2}\dfrac{FL^3}{EI}, \dfrac{4}{3}\dfrac{FL^3}{EI}$ F) $-\dfrac{1}{3}\dfrac{FL^3}{EI} + \dfrac{FL^3}{GI_p}, \dfrac{1}{2}\dfrac{FL^3}{EI}, \dfrac{4}{3}\dfrac{FL^3}{EI}$

Formulae

Second moments of area

Rectangular cross-section

$$I = \frac{1}{12} b h^3$$

Triangular cross-section

$$I = \frac{1}{36} b h^3$$

Hollow circular cross-section

$$I = \frac{1}{4} \pi (R_u^4 - R_i^4)$$

Polar moment of area

$$I_p = \frac{1}{2} \pi (R_u^4 - R_i^4)$$

Stress

$$\sigma = \frac{N}{A}$$

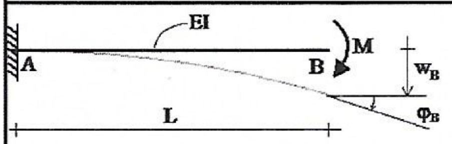
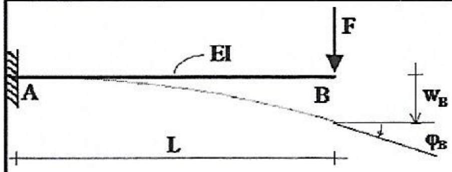
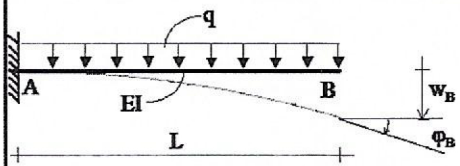
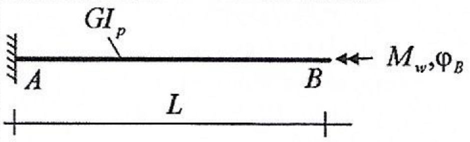
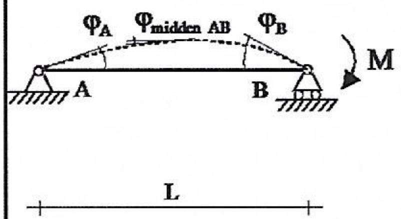
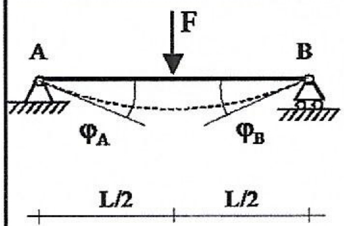
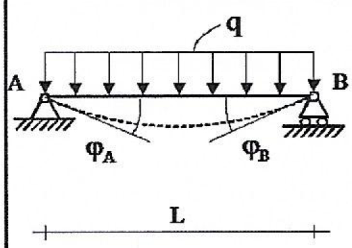
$$\sigma = \frac{M z}{I}$$

$$\tau = \frac{V S}{b I}$$

$$\tau = \frac{M r}{I_p}$$

Elongation

$$\Delta l = \frac{N l}{E A}$$

Standard beam equations	
	$\varphi_B \circlearrowright = \dfrac{ML}{EI}$ $\quad w_B \downarrow = \dfrac{ML^2}{2EI}$
	$\varphi_B \circlearrowright = \dfrac{FL^2}{2EI}$ $\quad w_B \downarrow = \dfrac{FL^3}{3EI}$
	$\varphi_B \circlearrowright = \dfrac{qL^3}{6EI}$ $\quad w_B \downarrow = \dfrac{qL^4}{8EI}$
	$\varphi_B = \dfrac{M_w L}{GI_p}$
Additional standard beam equations	
	$\circlearrowleft \varphi_A = \dfrac{ML}{6EI}$ $\quad \circlearrowleft \varphi_{middle\_AB} = \dfrac{ML}{24EI}$ $\varphi_B \circlearrowright = \dfrac{ML}{3EI}$ $\quad \uparrow w_{middle\_AB} = \dfrac{ML^2}{16EI}$
	$\varphi_A \circlearrowright = \dfrac{FL^2}{16EI}$ $\quad \circlearrowleft \varphi_B = \dfrac{FL^2}{16EI}$ $\downarrow w_{middle\_AB} = \dfrac{FL^3}{48EI}$
	$\varphi_A \circlearrowright = \dfrac{qL^3}{24EI}$ $\quad \circlearrowleft \varphi_B = \dfrac{qL^3}{24EI}$ $\downarrow w_{middle\_AB} = \dfrac{5}{384}\dfrac{qL^4}{EI}$